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A

TREATISE

OF THE

MECHANICAL POWERS,

WHEREIN THE

Laws of Motion,

AND THE

PROPERTIES

OF THOSE

POWERS

ARE

Explained and demonſtrated in
an eaſie and familiar METHOD.

Being the Subſtance of certain Diſcourſes delivered at the GEOMETRY-LECTURE *at* Greſham-College.

By ANDREW MOTTE,

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PREFACE.

22 a/p 22 S.S.
Q 7. 17. 35

THE Reader will meet in the Title-Page with an account of the Occaſion that gave birth to the following Treatiſe. Thus much farther it may not be amiſs to take notice of, that as it was written to be deliver'd by word of mouth only, the Writer thought it beſt to put what he had to ſay into a continued Diſcourſe, and not into the Form uſed by Mathematicians; making it his Endeavour to find a way of Expreſſion as familiar and approaching to common Speech as he could conveniently; aiming throughout at Perſpicuity rather than Conciſeneſs. This proving not unacceptable to ſome Hearers, it was imagined that it might chance to fall in with the Taſte of other perſons.

It may not be unfit to ſay a few words here concerning the Manner, in which this Treatiſe is diſpoſed. And firſt it *begins with the* Laws of Motion. *Not that this previous Step is abſolutely neceſſary, it being eaſie to have ſhewn the Properties of the Mechanical Powers without it. But the Importance of thoſe Principles being ſo great, and the Underſtanding of them being a convenient tho' not a neceſſary Introduction to the remaining part, I am perſuaded the Reader will not be diſpleaſed to meet them there. Their extenſive Uſe, as being the Foundation of all Natural Philoſophy, inclined me to treat of them more diffuſely than has hitherto been done; partly for an Introduction to higher Speculations, and partly becauſe it has been my fortune to meet ſometimes with ingenious perſons, but not converſant in theſe matters, who have not ſo readily underſtood and aſſented to ſome things relating to them, as one would have expected.*

From thoſe Principles the Properties of the Mechanical Powers are demonſtrated; in the doing of which, Mathematical Terms have been avoided as *much as could be done conveniently; and, where they were neceſſary, are explained. There is added at the End, a Diſcourſe concerning the Sail; which, tho' not uſually reckoned with the other Powers, is of too univerſal Uſe to be omitted. There is alſo ſubjoined an Appendix, in which ſome Propoſitions, which were unavoidably expreſs'd in Geometrical Terms, are exemplified in Numbers; which will be of ſervice to the ungeometrical Reader, by explaining the Meaning and Uſe of the Propoſitions, while he takes the Proofs upon Truſt.*

Beſides thoſe for whom this Work is immediately intended, who having already exerciſed themſelves in pure Geometry are willing to make ſome farther Progreſs in the Mathematical Sciences, I conceive it may be of ſome ſervice to the following Perſons. Firſt, to thoſe Gentlemen who have gone thro' Courſes of Experiments; becauſe as the Subject is here treated in the ſame way of Diſcourſe, in which the Experiments they ſee are explained to them, and not in a

Mathematical Form; they may refreſh their Memories with ſome things they have forgot or remember but imperfectly.

It may alſo be of uſe to thoſe who are juſt entring upon the Study of Natural Philoſophy, as they will find the Laws of Motion, *which are the Foundations of that Science, more largely explained than is commonly done; which, when thoroughly apprehended, will prove a great Advance to more difficult Speculations.*

There is another ſort of Perſons, whom I had in my eye, and would be very glad to benefit; which are induſtrious and dextrous Workmen. Of theſe I have known ſeveral, who would be well pleaſed to underſtand the Nature of thoſe Machines which they daily uſe, but underſtand no otherwiſe than by Uſe. Such Perſons will I believe be well contented to find the Reaſons here explained of ſeveral Things, which they do at preſent only becauſe they have ſeen them done before. I can but think, that with ſome Things they don't apprehend, they will find others that they may, and *which it will be both a Satisfaction and a Convenience to be informed of; the Things here explained being not dependent upon one another, but proved and handled in different manners.*

The Way I would therefore recommend to ſuch Readers, ſhould be to read the Whole through, paſſing over ſuch things as they don't well underſtand, and making the beſt Advantage of what they do. This Method of proceeding may invite a ſecond Review, in which they will find many things become eaſier than they appeared at the firſt Peruſal.

After all, I ſhall be very much obliged to the good nature of my Readers, if they find half the Conveniences I here point out to them; and I ſhall be ſtill more obliged to them, if for the ſake of my real Deſire of being ſerviceable, they will be ſo kind as to pardon my many Errors and Defects.

Nov. 30.
1726.

THE

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A TREATISE OF THE MECHANICAL POWERS.

SECT. I.

The LAWS *of* MOTION.

I SHALL begin with an Account of the Science of Mechanics in general, the Subject of it, and the Principles upon which it builds its Concluſions. Afterwards I ſhall deſcribe the ſeveral mechanical Powers in Order;

and ſhew their ſeveral Natures and Properties, deduced from thoſe Principles.

ALL Matter is indued with a Property of giving Motion to other Bodies and receiving it from them. This Property is ſo univerſal, that upon it all the Operations of Nature and Contrivances of Art depend; and by the Wiſdom of HIM *who created all Things in Number, Weight, and Meaſure*, it is made ſubject to Geometrical Reaſoning and Calculation; the Application of which to the Conſideration of Forces and Motion is the Buſineſs of the Science of Mechanics. Mechanics therefore is the Science, by which we diſcover the different Manners in which Forces may be applied to Bodies, and the different Effects produced thereby in the Bodies, as to Velocity, Direction, and Quantity of Motion.

NOW this Science, being Part of what is called the mixed Mathematics, requires ſome Axioms or general Principles, found out by certain and indubitable Experience, joined with ſtrict Geometrical Reaſoning, whereon to found its Concluſions.

AXIOM I.

EVERY Part of Matter in Motion, or at Reſt, continues at Motion, or at Reſt, till ſome external Cauſe alter it. Firſt, that Bodies at Reſt continue at Reſt till ſome outward Cauſe diſturb them, is plain from Experience. A Stone lying upon the Ground will lie there continually, without moving either ſideways or upwards, till ſomething elſe ſtir it. This Obſervation being found true of ſuch Bodies as come under our Notice, we infer the ſame of thoſe Bodies we can't come at. And this is confirmed alſo by Reaſoning. For a Stone upon the Ground, if it give Motion to it ſelf, muſt cauſe it ſelf to move in ſome given Direction. Say it be to the right; but for as good a Reaſon it may as well be ſaid to move to the left; but it can't move both Ways at the ſame time, therefore neither. The Heavineſs of

Bodies, or the Force with which they deſcend towards the Earth, when placed at Reſt in the Air, I take no Notice of; that being certainly occaſioned by an outward Force, tho' the Cauſe be not perceivable by any of our Senſes.

THE ſecond Part of this Axiom, that Bodies in Motion will continue in Motion till ſome outward Force alters their State, is not indeed plain from Experience as the former is; becauſe all Bodies here upon Earth put into Motion will ſoon ceaſe to move, ſome for one Cauſe, and ſome for another, unleſs a new Force be impreſs'd on them. But to thoſe Experiments it is anſwer'd, that we can aſſign ſuch Cauſes, why thoſe Bodies ceaſe to move, as plainly ſhew that this Axiom may nevertheleſs be true. And theſe Cauſes are the Reſiſtance of the circumambient Air, the Force of Gravity, and the Friction of Surfaces; and that theſe Cauſes are the true ones appears from hence, that the more theſe are diminiſhed, the longer the Motion continues; whence it is reaſonably concluded, that if they could be entirely removed, it would not ceaſe at all. This is made much more plain by Reaſoning. For there can be no Reaſon given, why a Body in Motion ſhould ſtop of it ſelf at one Place or Inſtant, rather than another; any more than why a Body at Reſt ſhould move of it ſelf at one Inſtant, or in one Direction, rather than another. Let a Body as

A be impelled along the Line A B C D; I ſay, that all outward Reſiſtance being taken away, it will move for ever in that Line. For if you ſay it will ſtop any where, aſſign the Point where it will do ſo. Let us for Argumentation ſake ſuppoſe that Point to be C. But why at C, any more than at B or at D? The Cauſes of Reſiſtance from without are taken away, and what can operate upon the Body to make it ſtop there, rather

than

than at B ? Or what can hinder it from going on farther to D ? If you conſider, you will find there can be no Reaſon given for its ſtopping at C, rather than at B ; for by our Hypotheſis, it meets with no more Hindrance to its Motion between B and C, than it does between A and B. Therefore if you ſay it ſtops at C, for the ſame Reaſon you may ſay it will ſtop at B, but it can't ſtop at both ; therefore at neither. In the ſame manner, we have prov'd it will not ſtop at C, one may alſo prove it can't ſtop at D ; and ſo on *ad infinitum.*

HAVING now ſhewn, that every Body or Part of Matter in Motion, or at Reſt, remains ſo till ſome outward Force alters its State ; the next Thing is to take notice what thoſe Forces are, which in Practice and Experience are made uſe of to that purpoſe ; and thoſe are, *Firſt*, The Colliſion of ſome Body that is in Motion with one that is at Reſt. *Secondly*, The Application of the Strength of ſome animated Creature to the Body that is to be moved, or to ſomething contiguous to it.

Now in order to limit, and determine the Quantity of theſe Forces, that we may handle and conſider them mathematically, they are expreſs'd either by the Velocity they produce in a Body impell'd by them, the Weight they are able to ſuſtain or move, or by both compounded together. The Velocity of a Body is meaſur'd by the Quantity of Space it goes over in a given time; if that Quantity of Space be great, the Velocity is ſaid to be great; if little, the Velocity is ſaid to be little.

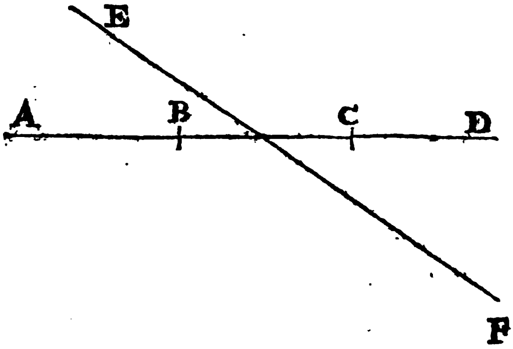
If we then imagine the ſame Body to be at one time ſo impell'd as to move 15 Foot in a Second of Time, and at another time ſo as to move 30 Foot in a Second, we expreſs theſe two different Forces by ſaying, that it was impell'd in the former Caſe by a Force that is as one, and in the latter Caſe by a Force that is as two. A Force is alſo valued by the

Weight it is able to ſuſtain; as ſuppoſe upon one Arm of a Ballance there hangs a Weight of 20 Pounds, if a Man preſſing with his Hands upon the other Arm keeps the Weight barely ſuſpended; we ſay, he applies thereto a Force of 20 Pounds. A Force is alſo valued by the Rectangle, or Product of theſe two, the Velocity, and the Weight of a moving Body, which is called the Momentum of that Body. If a Man throws out of his Hand a Stone of 2 Pounds Weight, with a Velocity of 20 Foot in a Second of Time, the Force, with which at its going out of the Hand it would ſtrike any Body in its Way, is called the Momentum; which we ſay, is as 2 × 20. If the ſame Man ſhould after throw a Stone of 3 Pounds, with a Velocity equal to 40 Foot in a Second, the Momentum would be as 3 is to 40; that is, 120. In theſe Caſes we ſhould ſay, The different Forces with which thoſe Stones were thrown are as 40 to 120, that is, as 4 to 12; which is the ſame as 1 to 3.

THIS being premiſed to ſhew the Method by which we value and expreſs the Quantity of Force apply'd to a Body, there remains another thing to be conſider'd in a Force, and that is the Direction of it. Now the Effects of Forces are made very different by their various Directions; and therefore they require a particular Conſideration; the whole Doctrine of Projectiles, which comprehends the Art of Gunnery, and the Knowledge of a good Part of the Phyſical Aſtronomy, depending upon the Inveſtigation of the Paths, which Bodies will deſcribe in Conſequence of Forces apply'd in given Directions.

I ſhewed above, that if a Body were impell'd by a ſingle Force, and no Reſiſtance were made to it, it would move for ever in the right Line prolong'd, in which the Center of it at firſt mov'd. That Line is called the Line of Direction of its Motion, and is always given by Poſition, when we would argue concerning Propoſipoſitions relating to it; that is, its Situation with reſpect to ſome given Place muſt always be determin'd. A Body may be, indeed, impell'd by more Forces than one, and in different Directions at the ſame time, the Conſequences of which it is the Buſineſs of this Science to determine, as we ſhall hereafter take notice. But for the preſent we ſhall only obſerve, that as the Velocity with which a Body moves is capable of infinite Variety; ſo likewiſe is the Line of Direction. A Body may be ſuppoſed to move 10, 100, 1000 Foot in a Second of Time, or if you pleaſe as many Miles; ſo the Line of Direction of its Motion may be as various as the Inclinations of Lines to each other are. As the one is capable, by the Addition of a new Impulſe, to be alter'd as to Quantity; ſo is the other equally liable by the ſame Means to be alter'd as to its Poſition. If a Body as A be moving along the Line

A B



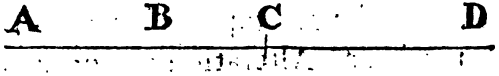
A B C D, with a Velocity able to carry it from A to B in a Second of Time, it may, by a new Force apply'd to it in the ſame Direction, receive a Velocity able to carry it as far as from A to C in the ſame time. And in the like manner, it may by a Force apply'd in a different Direction, as for Example, along the Line E F, receive an Alteration not only as to Velocity, but as to its Direction alſo; ſo as to make it move in a different Line from that along which it was at firſt impell'd. That we may, therefore, argue upon ſolid Grounds concerning theſe Mutations, I ſhall proceed now to the ſecond Axiom.

2. THE

AXIOM II.

The Increaſe or Decreaſe of Motion, produced in a Body by a new Force apply'd to it, is as to Quantity proportionable to the Quantity of this new Force; and the Change of the Motion as to Direction is made towards the Direction of this new Force.

I Say the Increaſe or Decreaſe of Motion, for a new Force apply'd to a Body in Motion may leſſen or ſtop its Motion inſtead of encreaſing it. Suppoſe a Body be moving along the Line A B C D, with any given Velocity from A to D;



if a contrary, and equal Force ſhould meet it in any Point, as C, whoſe Line of Direction is from D to A, inſtead of increaſing its Velocity, it would entirely ſtop its Motion. As is very plain, for the Body can only move either from A towards D, or elſe from D towards A;

for there being no other Force impreſs'd on it, it can move no other Way. But it can't move towards A, becauſe the Force directed from A to D hinders it; nor from A towards D, becauſe the oppoſite and equal Force reſiſts it; therefore it won't move at all. And if we ſhould ſuppoſe this new and contrary Force to be but half, or one third ſo great as the former, it is eaſy to conceive it will diminiſh its Motion or Velocity, tho' it will not entirely ſtop it.

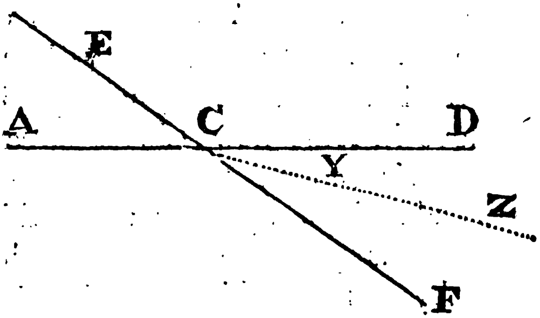
THIS Increaſe or Decreaſe of Motion is always proportionable to the Force that produces it; ſo that if the new Force conſpire with the former, having the ſame Direction, it will add to the Motion; if it be contrary, it will diminiſh it. As if in the ſame Figure a Body be moving with a given Velocity along the Line ABCD, if a new Force be applied to it in the Point C, having the ſame Direction, and double to the former, it will ſuperadd a double Velocity to that with which it moved before.

The new acquir'd Velocity will then be to the former, as 1 to 1 + 2, that is, as 1 to 3; there being a Velocity, which is as 2, added to the former, which is as one.

BUT if this new Force, which is as 2, were to move in a contrary Direction to that in which the Body is moving with a Velocity that is as one, it would then add to it a negative Velocity as I may call it = 2; that is, it would repel it with a Force = 2. Its former Velocity would then be to its latter as 1 to 1 — 2, that is, as 1 to — 1; that is, it would move back from C to A with a Velocity equal to that with which it mov'd before from A to C. The Rule then to know the Velocity with which a Body moves after a new Force is apply'd to it, is to add the Values of the two Forces together; making the new Force affirmative if it have the ſame Direction as the former, and negative if the contrary.

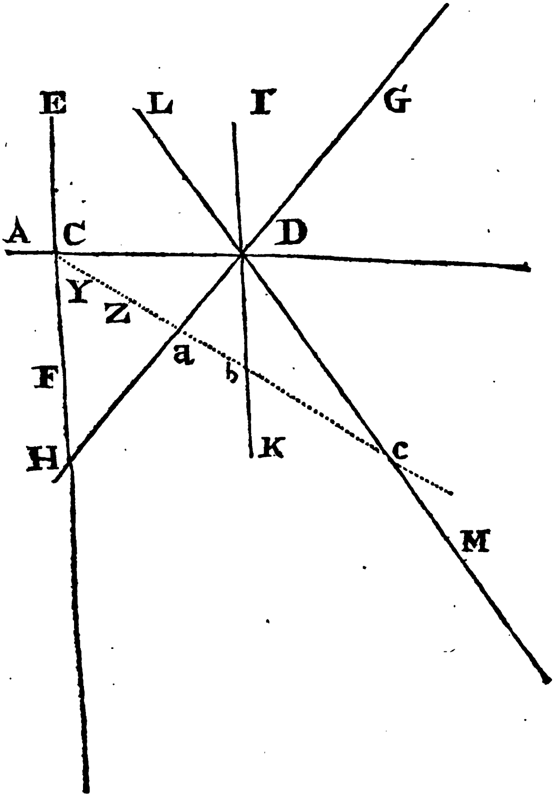
BUT we have hitherto only proceeded upon the Suppoſition, that the new

Force is directed along the ſame Line in which the former was; but this is very far from being always the Caſe. For it commonly happens, that the new Force is applied along a Line, which makes an Angle with that in which the former Force is applied or directed; as in the Fig.



Here the Body, having mov'd along the Line A C D as far as the Point C, is there impell'd by a new Force, whoſe Line of Direction is the Line E F. It will then be proper in the next Place to enquire what the Conſequence will be of this new Force. And this Enquiry is anſwered by the ſecond Part of this Axiom, to wit, that there will be an Alteration of the Line of Direction, which will be made towards the Line of Direction of this new Force, that is towards the Line E F. I ſay towards the Line E F, for it can't move along it, being hinder'd by the Force A D impelling it another Way; and much leſs will it be able to get into any Line without E F. Since therefore it cannot move along A D by Reaſon of the Force E F, nor along E F by Reaſon of the Force A D, it follows, that it will move in ſome new Line between them as Y Z; and by the firſt Part of this ſecond Axiom, by how much this new Force is greater or leſs than the former, by ſo much this new Line Y Z will be nearer to or farther from the Line E F.

IN order to ſmooth the Way to an important Propoſition, which I ſhall deduce from what has been ſaid, let there be drawn as in the Figure

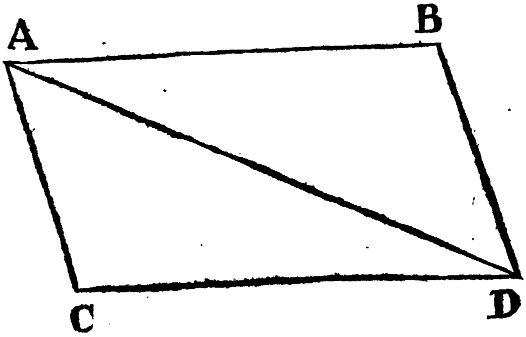


the Line A C D, along which the Body A is impell'd by a Force directed from A to D. Let it be impell'd in the Point C by a new Force, whoſe Line of Direction is E F. Let there be drawn thro' the Point D the Line G H *inclining towards* the Line E F, and meeting it in H; the Line I K thro' the ſame Point *parallel to* the ſame Line E F; and the Line L M thro' the ſame Point, *declining from* the Line E F. It is plain, that if this new Force E F were not applied, the Body A impell'd by the Force A D alone, would paſs thro' theſe three Lines at the ſame Inſtant, that is, at their common Section the Point D. But when by the Application of this new Force it is obliged to alter its Path or Direction, and to move in the Line Y Z as it was juſt now prov'd it would do; it is plain, that it will interſect theſe three Lines ſucceſſively, as at the Points a, b, c, at different Inſtants of Time. Now I ſay, that in this Caſe it will touch the Line G H in the Point a, in leſs Time than it would have touch'd it in the Point D, if it had been impell'd only by the ſingle Force A D. As is very plain; for here are two Forces conſpiring together in impelling it towards that

Line, one towards the Point D, and the other towards H, which muſt neceſſarily impart a greater Degree of Velocity, than either of them ſingle. On the other Hand it will be longer before it reaches the Point c in the Line L M, than in moving to D in the former Caſe. For the Force E F, as one may ſee only by Inſpection of the Scheme doth now continually tend to draw it *from* the Line L M, and for that Reaſon muſt undoubtedly retard its Motion towards it. Since then the Body will arrive ſooner at the Lines inclining towards E F as G H, and later at thoſe declining from it as L M; it follows that it will meet a Line in the intermediate Poſition, as I K which is parallel thereto, exactly at the ſame time in b, that it would have met the ſame Line in the Point D, had it been impell'd only by the ſingle Force A D. For ſince on the one Side of I K, the Time is leſs, and on the other Side more, it follows, that in I K it ſelf, the Times muſt be equal; that is, the Body A ſo impell'd by ſuch Force, will be found in a Line parallel to the Direction of that Force, in the ſame Time (tho' not in the ſame Point) as it would have been, if that Force had not been apply'd. I ſhall ſhew the Uſe of this in the Demonſtration of the following Propoſition.

If two Forces in different Directions be apply'd to a Body, it will deſcribe the Diagonal of a Parallelogram, along whoſe Sides thoſe two Forces are directed, and in the ſame given time in which the Body would deſcribe either of thoſe Sides.

Let then a Force be apply'd in the



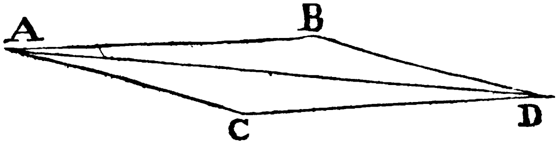
Point A to a Body in that Point; and let that Force be of ſuch a Quantity as to be able to impel it as far as B in a given Time. Let at the ſame Inſtant another Force be apply'd along A C ſufficient to carry the Body to C in the ſame given Time. Compleat the Parallelogram by drawing B D and C D; I ſay, that the Body will move along the Diagonal A D of this Parallelogram, in the ſame Time in which it would have mov'd along either of the two Sides, A B or A C, by the Means of either of thoſe Forces ſingle. For the Force A C being parallel to the Line B D, we have juſt now prov'd, that it will neither accelerate nor retard the Motion of the Body to the Line B D, ſo that it will be found in the Line B D in the ſame Time exactly, that it would have been if that Force A C were not applied to it. And ſo in like Manner the Force A B being parallel to C D, will not at all hinder, but that the Body A impell'd by the Force A C will arrive at C D, exactly in the ſame Time as it would have done, if that Force A B had not been apply'd at all. It will therefore be found ſomewhere in the Line C D, at the ſame Inſtant of Time, that it will be in B D; therefore it will be in the Point D, their common Interſection. In this manner the two Forces A B impelling the Body towards B, and A C impelling it towards C, will be compounded into a ſingle one impelling it from A to D. But a Body impell'd by a ſingle Force will proceed in a right Line; therefore along A D the Diagonal of the Parallelogram A B C D.

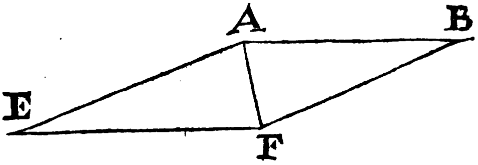
I have been the longer upon this Propoſition, and ſpent the more Time and Words, in explaining and making eaſy the Demonſtration of it, becauſe of its abundant and univerſal Uſe, which extends it ſelf to the Solution of almoſt every Problem, and the Diſcovery of almoſt every Truth, which relates to the Motion of Bodies impell'd by outward Forces, in what manner ſoever apply'd. The whole Doctrine of Projectiles, which comprehends the Art of Gunnery, is built upon this Prop. for by the Application thereof it is demonſtrated, that all Bodies projected by any Force with a Rectilinear Direction, near the Surface of the Earth, will by the Power of Gravity continually acting upon them, be obliged to deflect from that Direction, and move in the Curve of a Parabola.

BY the help of this Prop. alſo, Sir *Iſaac Newton* in his *Principia*, has demonſtrated the true Syſtem of the World, made manifeſt the Laws by which the heavenly Motions are directed and regulated, and the Effects and Influence the Celeſtial Bodies have upon each other; and by this Means has clear'd ſeveral Difficulties, and made Diſcoveries relating to them, that were thought inacceſſible before. It is alſo uſeful to give an Idea of the Generation of Curve Lines, which it does eaſily and naturally, and by that Means ſerves to clear the Way to the underſtanding the Nature of Fluxions. And laſtly, it is ſerviceable in demonſtrating the Nature and Properties of the Mechanical

Powers, with great Brevity and Facility. At preſent we ſhall deduce ſome Concluſions from thence, applicable to the preſent Subject.

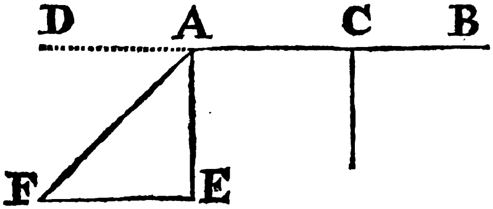
1. FROM hence we may deduce the Compoſition of two given Forces into one, as alſo the Reſolution of any one ſingle Force into an infinite Number of compound ones. By the firſt we are able to find out the Change as to Velocity and Direction which is made in a Body by a new Force impreſs'd thereon, and to aſſign the Degree of Velocity, which is added to, or taken away from a Body in Motion by a contrary oblique Force.





IF a Body be moving along A B with a Velocity able to carry it from A to B in a Minute of Time, and one would know how much additional Velocity it would receive by an oblique Force A C able to carry it from A to C in the ſame time; 'tis plain, that it is only neceſſary to compleat the Parallelogram, and to draw the Diagonal A D; and the Exceſs of the Diagonal above the Side A B is the additional Velocity ſought. So again, if one ſhould ſuppoſe the Force A E applied inſtead of A C, 'tis plain, that this Force has a Direction contrary to that of the Force A B, and therefore will take from its Velocity inſtead of adding to it; in this Caſe we muſt proceed as before, and drawing the Diagonal A F, the Exceſs of the Side A B above the Diagonal, will give the Quantity of Velocity loſt by Means of this new Force.

2. By this Prop. we are able to reſolve a ſingle Force into two, and thereby to know the Effect it will have when applied in a given Manner. Suppoſe the



Line A C B to have one Point of it at C fix'd, like the Center of a Circle; and the other Parts to be capable of turning round that fix'd Point, like the Radius round the Center, by a Force applied to it in any Point as A or B, either puſhing forwards, or pulling backwards. Now it is plain, that if this Force be applied exactly along the Line A C B, it will have no Effect at all in making the Line revolve about its Center, whether the Force puſh forwards towards C, or pull backwards towards D. But on the contrary if we ſuppoſe this Force apply'd at right Angles to A C B, or along A E, whether puſhing upwards towards A, or downwards towards E, it is plain, that it will have a very great Effect, as being directly contrary to the Poſition of the Line A C B, and therefore capable of making a very great Alteration in its Situation. Now ſince this Force may be apply'd in all manner of Directions between that of A E and that of A D, ſome of which Applications will have more Effect than others: We make uſe of this Prop. not only to know which is the moſt effectual of all, but the exact Quantity and Proportion of the Effect of each of theſe various Applications.

We have ſhewn before, that a Force is valued and expreſs'd by the Velocity it produces in a given Time; and that Velocity by a given Line deſcrib'd with that Velocity in that given Time: One may therefore make uſe of a Line to expreſs a given Force, in Proportion to the Length of which Line the Force may be valu'd. If we ſhould ſuppoſe then the Line A E to expreſs the Quantity of a given Force; I ſay, that Force apply'd at right Angles will have an Effect equal to a greater Force apply'd obliquely. For by the laſt Prop. if we draw the Line E F perpendicular to A E and join A F, we ſhall reſolve the Force A E into two, E F and A F, which two will by the ſame Prop. have the ſame Effect as the other ſingle one. Now the Force E F being parallel with the Line A C, can have no Effect at all, as was juſt now ſhewn; therefore the Effect of A F alone is juſt equal to the Effect of A E. But the Line A F lying over againſt the right Angle is longer than A E, and therefore the Force, which is proportionable to it muſt be greater; therefore a Force apply'd at right Angles will have an Effect equal to that of a greater Force apply'd obliquely. By the ſame Method one may eſtimate the Quantity of Force that is loſt by an oblique Application. For ſuppoſe in the ſame Fig. the Force A F is apply'd, it is plain by what is juſt ſaid, that ſome of that Force is loſt; becauſe if it had been applied at right Angles it would have had a greater Effect. In order therefore to know the exact Quantity, reſolve this Force into two A E and E F, and the Quantity of Force that is uſeleſs by the oblique Application of it, will be to the whole, as E F to A F.

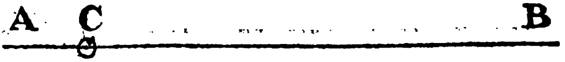
I have hitherto conſider'd the Effects of Forces ſimple or compound upon a Body, as to the various Quantities of Velocity they will impreſs upon it, and the various Lines of Direction in which they will cauſe it to move, according to the infinite Variations of Quantity and Direction, which Forces are capable of receiving.

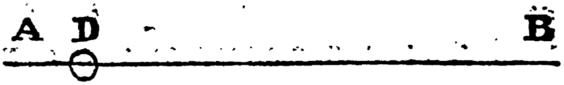
But in this we have gone all along upon the Suppoſition, that theſe Effects are produced in the ſame or equal Bodies, ſuppoſing the ſame Force to receive no Alteration as to its Effect from the Body it is apply'd to. But in conſidering the Effect of Forces upon Bodies there is another Thing to be reſpected, and that is the Quantity of Matter contain'd in the Body. As we have then hitherto been diſcovering the Effects of Forces upon Bodies, where the Bodies have remain'd the ſame, and the Forces have differ'd; ſo we ſhall now conſider thoſe Effects where the Bodies are the ſame, and the Forces remain unalter'd, in order to lay the Foundation of our Reaſoning in Caſes where neither of them is the ſame.

Now the Difference of the Effects of Forces proceeding from the different Circumſtances of the Bodies takes its Riſe from the different Quantity of Matter,

ter contain'd in each. It will be therefore proper to ſpend a few Words in explaining what is meant by that Expreſſion, *The Quantity of Matter.*

EVERY ſolid Body may be divided into a given Number of equal Parts, which Number is proportionable to what the Mathematicians call the ſolid Content of a Body of that Figure. If we ſuppoſe a ſolid Cube, whoſe Side is 8 Inches, it is plain, that Cube may be divided into many equal and leſſer Cubes of 1 Inch, by multiplying according to the Rules of Geometry, that Side into it ſelf firſt, and afterwards into the Product of that firſt Multiplication. This gives 512, which is the Number of ſuch little Cubes whoſe Side is one Inch, as taken together compound the great Cube whoſe Side is 8 Inches, as is eaſily known by Geometry. But by the Rules of Geometry we alſo learn, that a Cone, a Pyramid, a Cylinder, a Sphere, or a Body of any irregular Figure may be aſſigned, that ſhall have the ſame ſolid Content as this Cube above-mention'd, that is, whoſe ſolid Parts taken together, will be equal to 512 of thoſe little Cubes. We ſhould ſay then, that ſuch a ſolid Sphere or other Body will contain in it ſelf an equal Number of ſolid Parts with that Cube; or in the Expreſſion we are here explaining that it contains the ſame Quantity of Matter. By the ſame Method one may divide that Cube into an immenſe Number of ſmall ones, whoſe Sides ſhall not contain above $\frac{1}{1000}$ Part of an Inch; an equal Number of which ſmall Cubes ſhall alſo be exactly contain'd in a ſolid Sphere of equal Content with the Cube. We may alſo imagine two ſolid Cubes, one which ſhall have twice the ſolid Content of the other, in which Caſe it will have twice the Number of equal ſolid Parts, that is twice the Quantity of Matter. The ſame may be alſo ſuppoſed of two Spheres or of any other Bodies, concerning which we ſhould ſay, that the greater Sphere has twice the Quantity of Matter in it that the leſſer has.





If we then ſuppoſe two Spheres impell'd along A B and A B by two equal Forces, and that the Sphere C hath but half the Quantity of Matter in it that the Sphere D hath; it is, I think, eaſy to diſcover, that the leſſer will move with twice the Velocity of the greater.

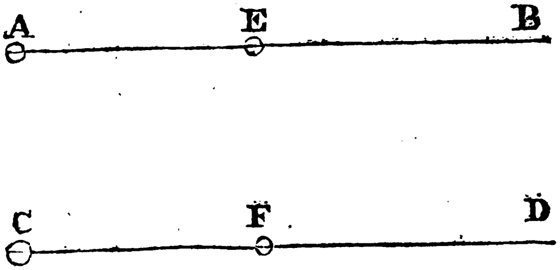
For if we ſhould ſuppoſe them both divided into a Number of equal ſolid Parts, it is plain from what was ſhewn juſt now, that the Sphere D will contain twice the Number of thoſe equal Parts that the Sphere C does. Let us ſuppoſe the Sphere D to contain 1000 of thoſe equal Parts, and the Sphere C 500. The Force therefore which impels D will be divided equally into

1000 Parts, and the Force which impels C into 500. Theſe Forces being at firſt ſuppoſed equal, it is evident, that one of thoſe equal Parts into which the Force at C is divided, is twice as great as one of thoſe equal Parts into which the Force at D is divided. For if we ſuppoſe two equal Lines to be divided one into 1000 equal Parts, and the other into 500, without doubt the 500th Part of one equal Line will be twice as great as the one 1000th Part of the other equal Line. Therefore each of the 1000 Parts of the Sphere D, will receive a Force but half ſo great as each of the 500 Parts of the Sphere C. If therefore every Part of the Sphere D receive but half the Force, every Part thereof will for that Reaſon move but with half the Velocity that every Part of the other does; but the Velocity of the whole Sphere, is the ſame with the Velocity of its Parts, therefore the whole Sphere D will move but with half the Velocity that the whole Sphere C does. Which was the thing to be prov'd. And ſince there can be no Pretence of

Reaſon for doubting or imagining that it will prove otherwiſe in any like Caſe; we draw this general Concluſion,

THE Velocities with which unequal Bodies move after being impell'd by equal Forces are reciprocally as the Quantities of Matter in each Body. The Meaning of which is, the greater the Quantity of Matter is, the leſs will the Velocity be in each.

BUT tho' the Velocities with which unequal Bodies impell'd by equal Forces are unequal, the Quantities of Motion are equal however. The Quantity of Motion is the ſame that I call'd before the Momentum, and is compounded of the Velocity with which the Body moves, and its Weight, which is always proportionable to the Quantity of Matter; and it is eſtimated by the Force which it is able to communicate to any Body that it meets with in its Way. Therefore unequal Bodies impell'd by equal Forces, tho' indeed they will not move with equal Velocities, will yet communicate the ſame Force to any Body, that either of them ſhall happen to meet with. That is if a Body A, whoſe



Quantity of Matter is as one, ſhall be impell'd along one Line A B, and a Body C, whoſe Quantity of Matter is as 4 be impell'd along C D, the Velocities with which they will move, will by the former Propoſition be unequal; that is, the Velocity of the great one will be ſo much leſs than that of the leſſer, by how much the Quantity of Matter therein exceeds the Quantity of Matter, or the Weight (which is the ſame thing) of the other. But its Quantity of Matter is 4 times greater, therefore its Velocity will be 4 times leſs.

YET I ſay, that if theſe two Bodies ſhould meet in the Points E and F, two new Bodies E and F at Reſt in thoſe Points, they will impreſs two equal Forces upon thoſe Bodies, notwithſtanding the great Difference there is of the Weight on one Hand, and of the Velocity on the other. The Reaſon is, that theſe two Differences being proportionable, compenſate each other, that is, the Want of Velocity is compenſated by the Exceſs of the Weight in the greater Body, and the Want of Weight in the leſſer is compenſated by the Exceſs of the Velocity. To ſhew this, let us ſuppoſe the largeſt Body C to be divided into 4 equal Parts, then each of theſe 4 equal Parts will be equal to the leſſer Body A. Each of theſe 4 equal Parts will then move by our Hypotheſis with a fourth Part of the Velocity of the Body A, and ſince each of them is equal to it as to Quantity of

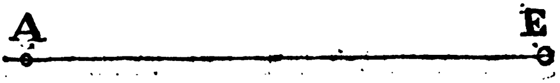
Matter, they will have each the ſame Effect upon the Body F, as the Sphere A would have upon the Body E, if moving with a Quarter of its Velocity: But the Sphere A has 4 times the Effect it would have, if it moved only with a Quarter of its Velocity, therefore it has four times the Effect upon the Body E, that one equal Part of the Sphere C has upon the Body F.

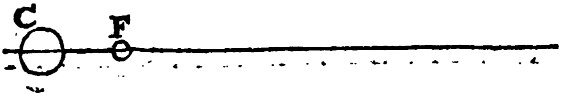
BUT if a fourth Part of the Body C, have a fourth Part of the Effect of the Body A, it follows, that the entire Body C will have the ſame Effect as the Body A exactly; that is, it will impreſs the ſame or an equal Force upon the Body F, as the Body A will do upon the Body E. Therefore if the Bodies E and F be equal, they will move after the Impulſe with equal Velocities; otherwiſe their Velocities will be in a reciprocal Proportion to the Quantity of Matter or the Weight of each.

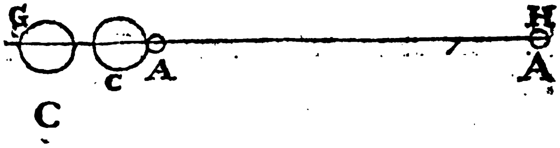
The Reſult then of our preſent Enquiry has amounted to the Diſcovery of theſe important Truths.

1. That the Velocities which two equal Forces impreſs upon two unequal Bodies, are reciprocally as the Quantity of Matter in each.

2. That notwithſtanding the Forces ſo impreſs'd upon the Bodies cauſe ſuch different Velocities, yet the Forces which thoſe Bodies are capable of communicating to another by impelling it, are equal; the Meaning of which is, that if we ſuppoſe two Bodies never ſo unequal as to the Quantity of Matter in them, yet if the Velocities with which they move be reciprocally proportional thereto, the Quantity of Motion in both will be equal; that is, they will impel any Bodies which they meet with in their Courſe with exactly equal Forces.

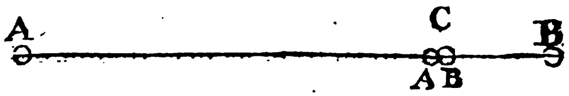






As for Example, if we imagine one Body A to be as one, and another Body C as 100, and to move with Velocities that are in the great Body as one, and in the leſſer Body as 100, they will impel any Bodies as E and F which they meet with in their Courſe with equal Forces. And for the ſame Reaſon if inſtead of moving along two ſeparate Lines, they ſhould move along the ſame Line as G H with contrary Directions, they will impel each other with the ſame equal Forces; and therefore ſince they urge one another with equal and oppoſite Forces they muſt both ſtop, as was ſhewn occaſionally before. And in the ſame Manner if they ſhould impel the two Arms of a Ballance at each End, they would impel thoſe two Arms with equal Forces, and therefore if the Ballance was in Equilibrio before, it will remain in the ſame Condition after the Impulſe. Thus you ſee that any two Motions will be equal between themſelves in which the Velocity of each Body multiplied into the Quantity of Matter or the Weight (which is the ſame Thing) produce two equal Rectangles or Products; and if they have oppoſite Directions will exactly counterpoiſe and ſtop each other.

AND for the ſame Reaſon if in any two oppoſite Motions, theſe Rectangles are unequal, the Body on whoſe Side the greateſt Rectangle is form'd, will prevail over the other, and impel it along the Direction in which it ſelf is moving.



As if A and B be moving along A C B with oppoſite Directions, and meet in C, and the Weight of A be as one, and its Velocity as three; and the Weight of B as two, and its Velocity as one. Multiply the two Numbers anſwering to the Velocity and Weight of each Body into each other, and in the Body A this Product will be three, and in B only two. Therefore the Body A will prevail over B, and there being a Force equal to 2 loſt on the Side of A, it will ſend B back again with a Force equal to one.

HENCE it appears, that the ſmalleſt Body in Motion is capable of moving the largeſt at Reſt. To make this Truth evident, which may ſeem ſurprizing at firſt, let us imagine a Body equal to one Pound Weight, moving with a Velocity of 3 Foot in a Second of Time; or which is the ſame Thing, ſuppoſe a Man with his Arm to apply a Force equal to it; I ſay that this ſmall Force would be able to remove, for Example, a Stone of 1000 Tun Weight that is entirely at Reſt, and not only to remove it, but to impreſs a Force upon it, that ſhall cauſe it to move as long and as far as he ſhall think fit, 100 or 1000 Yards, ſuppoſing all outward Reſiſtance to its Motion taken away.

For if we multiply on the Side of the ſmall Body its Weight one, into 3 its Velocity, it produces 3; but if we do the ſame on the other Side, the Velocity in that being equal to nothing, a Body at Reſt having no Velocity, it follows that if the other Number repreſenting the Weight be never ſo large, it being multiplied by 0 will produce no more than 0. Therefore the Rectangle on the Side of the greater Body being 0, and on the Side of the leſſer 3, it follows that the leſſer will overpower the greater, and cauſe it to move with a Velocity reciprocally proportional to the Weight. And this Force being once impreſs'd will by the firſt Axiom continue the Motion as far as one pleaſes, if all outward Impediments are remov'd.

BUT this Myſtery will eaſily be unfolded if we conſider that the Velocity of this Motion is reciprocally proportional to the Weight. Now the Weight being ſo large, the Velocity with which this great Body moves, muſt be exceeding ſmall; for a Stone of 1000 Tun Weight contains above two Millions of times the Weight of a Body of one Pound, and therefore muſt move above two Millions of times ſlower, which as one may find by an eaſy Computation, will not be above the rate of one Inch in 24 Hours. And in Practice that little Velocity will always be reſiſted and taken away by the Friction of Surfaces which was before taken notice of. So that tho' there be nothing more certain than this Truth, it is not eaſy to prove it by Experiment, by reaſon of accidental Hindrances. However to remove them as much as poſſible, there have been invented ſeveral practical Helps of different Kinds; ſome intended to avoid this Friction which is ſo great an Hindrance; others to collect and apply Force in a more convenient Manner than can be done without them, and theſe are called *Mechanical Powers.*

SECT. II.

Of the MECHANICAL POWERS.

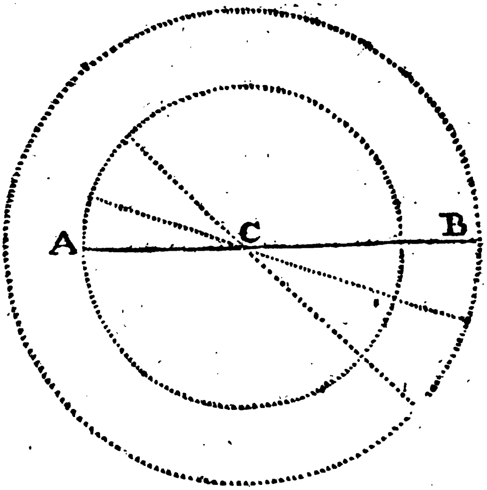
THE Mechanical Powers are commonly reckoned ſeven; the Ballance, the Lever, the Pully, the Wheel, the Wedge, and the Screw; to which ſome add the inclined Plane; but indeed the four firſt, and the three laſt, depend ſo evidently upon the ſame Principles, as might very well warrant the reducing of thoſe ſeven to two. However Cuſtom having order'd otherwiſe, and the Matter being of no great Conſequence, we may as well follow the old Reckoning. Theſe are alſo capable of being compounded together with great Advantage and Convenience, upon ſeveral Occaſions.

THERE are other Mechanical Powers beſides theſe; as the Sail of a Ship, and of a Windmill; the Spring, upon which the Force of a Bow in ſhooting an Arrow depends; the Battering Ram of the Ancients, the Hammer, and ſeveral others. But theſe ſeven above-mentioned being of more general Uſe, and admitting of greater Variety of Propoſitions relating to them, are moſt generally known by that Name, and conſider'd by Mathematicians. Only the Sail being become of ſuch univerſal and important Uſe, the Reader will not be diſpleaſed I believe, to have the Nature and

Properties of it diſtinctly explained; which he will accordingly find done in its proper Place.

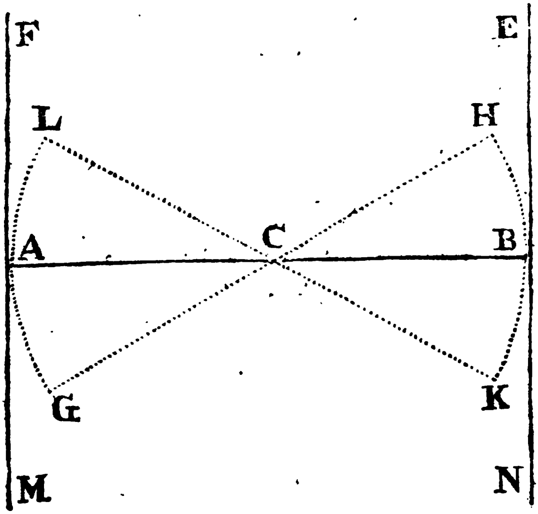
Of the BALLANCE.

EVery Body knows what a Ballance or Pair of Scales is in common Life, but in order to reaſon Mathematically upon it, we muſt look ſomewhat nicely into its Original.



IF in a Line as ACB we imagine the Point C fixed, and the two Segments thereof A C and C B revolving round that Center, like Radii of a Circle, the Points A and B will deſcribe Arches of Circles round that Point C, which ſhall be to each other as thoſe Lines. The Velocity alſo with which each Point of thoſe Lines moves, will be as the Arch that each Point deſcribes. For the Velocity is proportionable to the Space gone over; that is, to the Arch deſcribed. Now the Velocities being to each other as the Arches, and the Arches as the Circumferences, and the Circumferences as the Radii; it follows that the Velocities are as the Radii; that is, by how much the Line A C is longer than the Line C B, by ſo much the Velocity of the Point A is greater than that of the Point B; and tis evident that this will be the Caſe of all the other Points in either of thoſe Lines, that by how much their Diſtance is greater from the Point C, with ſo much the more Velocity will they move. And if any one ſhould ſuppoſe a Force of any Kind acting either at A or B, tis plain that let it make thoſe

Points move with what Velocity ſoever the Velocities of the other Points will ſtill be in the ſame Proportion; and that the Line A C B being imagin'd as perfectly inflexible, a Force apply'd to any one Point will be communicated to all the others on both Sides the Point C, in ſuch ſort as to make them move with Velocities proportionable to their Diſtances from the fix'd Point C. And this circular Motion once begun, will continue for ever by means of this Force, unleſs ſome new outward Force ſtops it, as was ſhewn of a Rectilinear Motion before. And as a Rectilinear Motion may be encreaſed by a Force conſpiring with its Direction, and leſſen'd or ſtopt by a leſſer or equal oppoſite Force, ſo in like Manner may this circular Motion be. And in like Manner as two Bodies impelled by Forces contrary and equal will exactly counterpoiſe each other in a Rectilinear Motion, where the Velocity and Quantity of Matter are on both Sides equal, ſo will they do in this. In the Fig. let there



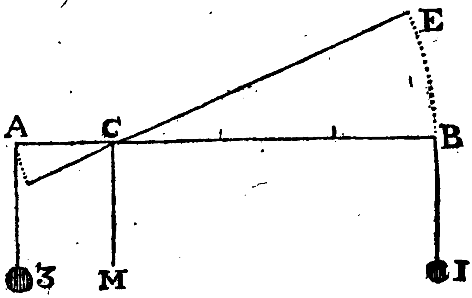
there be choſe the Point C in the Line A B equally diſtant from the Extremities A and B, and that Point C being fix'd, let the Lines A C and C B move round it with a circular Motion, and let two equal Bodies moving with equal Velocities along E B and F A, impel the Line A C B in A and B at the ſame Inſtant of Time. It is plain that the Force F A would of it ſelf cauſe the Line A C to revolve towards G, and the Line A C B being ſuppoſed inflexible, the other Segment C B muſt revolve towards H. So the Force E B would of it ſelf cauſe the Line C B to revolve the contrary Way towards K, and the Line A C for the ſame Reaſon towards L. Therefore theſe Bodies oppoſe each others Motion; and ſince they are equal as to Weight, Velocity and Diſtance from the Center, that Oppoſition on both Sides muſt be equal, therefore they can't turn the Line A C B round at all. And if we ſhould ſuppoſe the Strings M A, N B faſten'd at the Ends of the Line A C B, and that the Points A and B are pulled downwards E towards M and N by thoſe equal Bodies hanging at the Strings M A and N B, the Effect would ſtill be the ſame, and the Line A C B would remain in Equilibrio as before. Now this is the Caſe of the common Ballance, in which the two Scales are at equal Diſtances from the Center of Suſpenſion. For two Bodies of equal Weight put into thoſe Scales pull the Arms downwards, one on one Side, the other on the other. But by the

Make of the Machine, one Arm can't fall except the other riſes; and by reaſon of the equal Weights, each Arm is pulled downwards by a Force equal to that which raiſes it upwards; therefore the Arms can neither riſe nor fall, and conſequently the Ballance will remain in *Æquilibrio.*

THIS is the Caſe of the common Ballance; but there is another Kind of Ballance called the *Roman Ballance* or the *Stilliard;* in which both the Weights are unequal; and their Diſtances from the Centre of Suſpenſion unequal alſo. The Method in weighing any Thing with this Ballance is, to make the Diſtances of each Weight from the Centre of Suſpenſion reciprocal to the Quantity of the Weight; that is, to hang a Weight of one Pound 8 times farther from the Centre of Suſpenſion than the other of 8 Pounds; and ſo other Weights in the ſame Proportion. This is the Method in Practice: I ſhall now ſhew the Geometrical

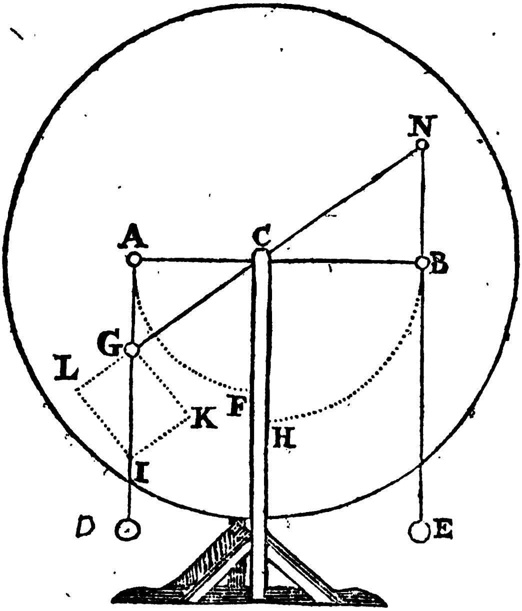
Reaſoning upon which this Practice is built.

IT was proved before, that two moving Bodies have equal Momenta or Quantities of Motion, when the Velocity of each Body multiplied into its Weight, produce on each Side equal Rectangles or Products. It was there ſhew'd alſo, that ſuch Bodies moving in the ſame right Line with oppoſite Directions will counterballance and ſtop each other. And that the Caſe was the ſame if they ſhould impel two Arms of a Ballance which are in *Equilibrio*. They would exactly counterpoiſe each other, and the Ballance would remain unalter'd. The Reaſon was ſhewn to be this plain one, that it requires the ſame Force exactly to impel one Part of Matter thro' three Parts of Space, as it does to impel three Parts of Matter thro' one Part of Space.



If we ſuppoſe then in the Fig. the Line ACB in a Horizontal Situation, and divided at the Point C in ſuch a manner that the Line AC may be to CB as 1 to 3; and that at the Point A there hang a Weight equal to 3, and at B, a Weight equal to 1, the Weights and Diſtances will be in reciprocal Proportion. In this Caſe then, the Line ACB will incline neither Way, but remain in the ſame Horizontal Situation it was in before. For at the ſame Time that the Point A pulled downwards by the Weight tends to deſcribe the Arch A D, the Point B being lifted up by the ſame Force tends to deſcribe the Arch BE. Now the Arches AD and BE being proportional to AC and CB will be as one to 3. Therefore the Weight at A which is as 3 tends to go over the Space A D which is as one, at the ſame Time that the Weight at B which is as one tends to go over the Space B E which is as 3. Now theſe two Tendencies or Efforts were ſhewn to counterpoiſe each other, and therefore ſince the two Weights tend contrary Ways with equal Forces, they can neither riſe nor fall but muſt remain in *Equilibrio.*

THIS Propoſition may be demonſtrated ſeveral other Ways; but the Method uſed occaſionally by Sir *Iſaac Newton* by the Reſolution of ſingle Forces into Compound is ſo very curious, that I believe it won't be diſagreeable to conſider that likewiſe, in a Manner ſomewhat more diffuſe, than was agreeable to his Purpoſe or Method.



Let us ſuppoſe in Fig. a circular Plane ſuſpended in a perpendicular Situation to the Horizon, by an Axis that paſſes thro' its Centre at right Angles to its Plane, the two Ends of which Axis being fix'd, the circular Plane may move round it, by a Force or Weight apply'd to any Point thereof. If there be hung upon any Point of this Plane as A, a Weight as

D, the circular Plane which before was in Equilibrio, by reaſon that all its Parts being at equal Diſtances from the Centre could not preponderate any way, will now by this adventitious Weight altering its Equilibrium, be forced to revolve round its Axis C, ſo that the Point A would deſcribe the Line A F. And ſo if we ſhould ſuppoſe the Weight E, hung at the Point B, by a String B E, it would make the Plane revolve the contrary Way from B to H. The Queſtion is then, what is the Situation and Quantity of two Weights, which having oppoſite Motions as D and E have, ſhall keep this Plane in Equilibrio as it was before. And I ſay, that if we ſuppoſe a Line A B drawn parallel to the Horizon, and the Weights to be ſuſpended upon that Line in ſuch ſort as that their Diſtances from the Centre C may be reciprocally proportional to their Weights, they will remain in Equilibrio.

BUT in order to demonſtrate this, we muſt premiſe the following Lemma, that the Effect of the Weight in turning this Plane will be the ſame in all Points of the Line AGID, as it will be in any one of them; that is, that it will have the ſame Effect at G or I, that it will have at A. This appears from what was before ſhewn of oblique Forces and their Effects, in *p.* 28.

LET us then ſuppoſe the Weights D and E hanging at A and B, and keeping the Plane in Equilibrio. Making the greateſt Diſtance CB Radius, imagine a Circle deſcribed upon the Plane that ſhall interſect the Line AD in G. Then will CG be equal to CB. Now by the Lemma juſt mention'd, the Weight D will have the ſame Effect pulling GC obliquely as the Weight E pulling CB at right Angles. In this Caſe if the Weights were equal, the Plane could not be in Equilibrio; for the equal Weight D acting obliquely, has leſs Force than the equal Weight E that acts at right Angles; ſo that the Weight E would overcome it and make the Plane revolve contrary, to the Suppoſition. Therefore to compenſate this Defect, its Weight muſt be ſo much the greater; that is, by how much the perpendicular Force is greater than the oblique Force, by ſo much muſt the Weight apply'd obliquely be greater than that which is apply'd perpendicularly. Now in order to find the Proportion of the oblique Force to the perpendicular, draw G K at right Angles to G C, and I K parallel to it, and compleating the Parallelogram L G I K, the oblique Force G I will be reſolv'd into two, G K and L G. Now by what I prov'd above the oblique Force will be to the Perpendicular as G K to G I. But the Triangles G I K and G I L are like as being formed by the Diagonal of the Parallelogram, and theſe two are like to the Triangle A G C; for the Angle A G C is equal to the Angle I G L, and therefore to G I K, and there is a right Angle in all the three; therefore all the three Triangles are alike or ſimilar. Therefore as G K to G I, ſo is A C to G C, but the oblique Force is in this Caſe to the Perpend. as G K to G I, therefore as A C to

G C, and G C being equal to C B, the oblique Force is to the perpend. as A C to C B. But it was ſhewn above, that by how much the perpendicular Force exceeds the oblique Force, by ſo much will the Weight apply'd obliquely exceed the Weight apply'd perpendicularly; that is as the oblique Force is to the perpendicular, ſo is the Weight E to the Weight D. But the oblique Force is to the perpendicular as A C to C B, therefore the Weight E is to the Weight D as A C to C B, which was the Thing to be proved.

Now if we ſuppoſe the uſeleſs and ſuperfluous Parts of this Machine taken away, there will remain nothing but the Line A C B, ſupported by a perpendicular Line at C, with the Weights D and E hanging at its Ends A and B, which is the Figure of the Ballance in our laſt Demonſtration. From hence we may infer,

1. SINCE two Weights hanging at the End of a Ballance will be in Equilibrio if they are in reciprocal Proportion of the Diſtances, ſo if they are not in that Proportion, they will not be in Equilibrio.

2. THAT on that Side where the Rectangle of the Weight into the Diſtance is greateſt, that Weight will prevail and cauſe the Ballance to deſcend on that Side.

3. THAT if we ſhould ſuppoſe the Ballance inclin'd to the Horizon as the Line G C N, the Weights will ſtill be in Equilibrio, if the Arms G C and C N be to each other in the ſame reciprocal Proportion as A C to C B; for it was prov'd before, that the Weight D pulling at G C had the ſame Effect as at A C; and for the ſame Reaſon, the Weight E will have the ſame Effect at N C that it has at B C. Therefore theſe two Weights have the ſame Effect; that is, keep the Plane in

Equilibrio as well when they act upon the inclin'd Ballance G C N, as when they act upon the Horizontal Ballance A C B. All that remains to be prov'd is, that G C is to C N as A C to C B. But the Angle A C G is equal to the Angle N C B; and there is a right Angle at A and B by the Conſtruction, therefore the Triangle A C G is ſimilar to the Triangle N C B. Therefore the like Sides having like Proportion, G C is to C N as A C to C B.

4. That if two unequal Weights are given, and it be requir'd to hang thoſe two Weights upon a Ballance, in ſuch manner, that they may keep the Ballance in Equilibrio, the Way is to divide the Length of the Ballance into two Parts that are to each other as the two Weights, and to hang the greateſt Weight at the end of the ſhorter Part, and the leſſer upon the longer.

5. That the leaſt imaginable Weight may counterpoiſe the greateſt, if the Ballance be divided in ſuch a Proportion as that the Diſtance of the fix'd Point from

either be reciprocally proportional to the Weight. That is, that a Weight of 1 Pound may keep ſuſpended a Weight of 100000, ſuppoſing the Diſtances from the fix'd Point to be reciprocally proportional to the Weight, that is as 1 to 100000. Theſe are the Properties of the Ballance conſider'd as ſuch.

BUT if we ſhould ſuppoſe one of the Weights taken away, and inſtead thereof any Force as of an animal Creature apply'd thereto, equal to the Force of the Weight, then the Ballance changes its Name, and is called a Lever.

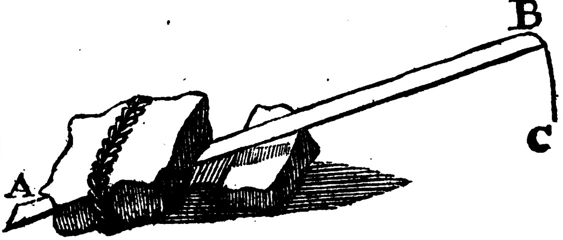
Of the LEVER.

I Obſerv'd above, that the ſmalleſt Body in Motion, or which is the ſame Thing, the ſmalleſt Force, is able to move the largeſt Body at Reſt, with a Velocity that is to its own in reciprocal Proportion to the Weight, and this is the moſt that can be done by human Power.

er. That is, a Man is able by the Laws which Nature has aſſign'd to Matter and Motion, if all outward Impediment and Oppoſition are taken away, by his own Strength alone to remove the greateſt Body, without having any Occaſion for the Aſſiſtance of any Machine. But the Matter is, that thoſe outward Impediments are ſo great, as to hinder this Law of Nature from taking Place. For Example, if a Man have Occaſion to lift a Weight of 40 *l.* from the Ground, and to ſet it upon any Place perhaps 3 Foot high; ſince this Weight is manageable by the Strength of moſt Men, it follows that he will have nothing elſe to do but to take it in one or both Hands, and to lift it to the Place deſign'd. In this Caſe it is plain, that the Weight will move with a Velocity equal to the Force; for a Weight that a Man holds in his Hand, will certainly move as faſt as his Hand does. But if it ſhould be requir'd to lift a Weight of 600 *l.* to the ſame Place, it was prov'd that he might do this alſo, if he could apply his Force to it in ſuch a Manner that the Velocity with which it moves, might be as much leſs than that of the former of 40 lib. as the Weight of 40 lib. is leſs than 600, that is 15 times leſs; if I ſay he could do this, he might remove this greater Weight alſo; only it being 15 times greater, it would by what we prov'd move 15 times ſlower. Now this can't be done without the Intervention of ſome Inſtrument or Machine, for as I ſaid, if a Man apply only the Members of his Body, the Force can move no faſter than the Weight does; but it is neceſſary that it ſhould be 15 times faſter; therefore ſome Machine muſt be applied. What thoſe are that are generally uſed for this Purpoſe, I mentioned before; the Lever, the Pully, the Wheel, the inclin'd Plane and the Screw. I am now to conſider the firſt of them.

THIS Inſtrument or Machine is generally in Practice a wooden or iron Bar, when uſed for the lifting of Weights, tho' it comes in Uſe under many different Forms upon ſeveral Occaſions of Life. There are four Kinds of Levers.

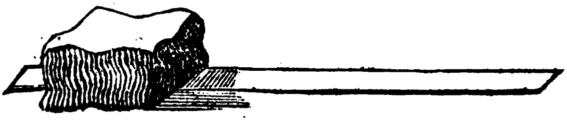
1. If a Man have occaſion to lift a great Weight, as for Example, a large and heavy Stone, entirely off the Ground, he makes uſe of a Lever of the firſt Kind. Having firſt got this Lever under the Weight that is to be mov'd, and faſten'd them together, he lays another Stone, or Body that will ſerve for a Prop, (or Fulciment, as it is often called) under his Lever, which Fulciment, for a Reaſon hereafter to be explain'd, he places as near the Weight that is to be mov'd, as conveniently he can. Then will the Lever, the Prop or Fulciment, and the Stone, appear as in the Figure.



This being done, with what Strength

he finds needful, he preſſes upon the longer End B of the Lever, downwards, in the Direction B C, which End B deſcending, makes the other End A of the Lever to aſcend, together with the Weight that is faſten'd to it. In this Caſe the fix'd Point is between the Power and the Weight.

2. IF a Man deſires to ſet ſuch a Stone, up on End only, without moving it entirely off the Ground, he makes uſe of a Lever of the ſecond Kind. Having got his Lever under the Stone, he lets it lie flat upon the Ground, without any Prop to ſupport it, as he did before. Then will the Lever and the Weight appear as in the Figure. Then inſtead of

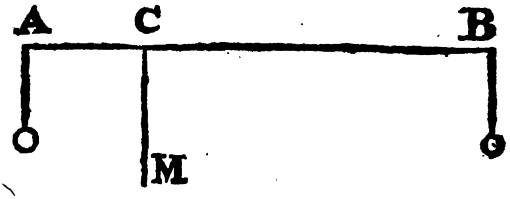


puſhing the Lever downwards as in the firſt Caſe, he heaves it upwards, ſo that the Weight lying upon one Part of it muſt aſcend at the ſame time with the Lever. In this Caſe the Weight is between the fix'd Point and Force or Power, the fixed Point being at the End of the Lever.

3. THE third Kind of Lever is where the Power is between the fix'd Point and the Weight. A Ladder or a May-pole rearing up may be conſider'd as a Lever of this Kind; that Part of the Ladder that is between the Man's Hands that raiſes it and the Top, being taken for the Weight, the Perſon's Hands for the Power, and the End of the Ladder that reſts upon the Ground for the fix'd Point.

4. THE fourth Kind of Lever is where the fix'd Point is at an Angle, and is called a bended or crooked Lever. A Hammer drawing out a Nail is ſuch a Kind of bended Lever. I come now to ſhew the Properties of theſe four Kinds of Levers.

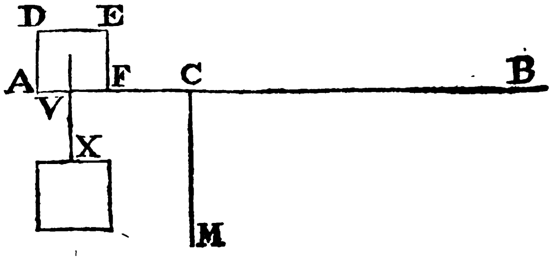
Now the firſt of theſe is very evidently the ſame thing, as the Ballance, where the Weights are unequal, and the Arms that ſupport them are unequal alſo. For if we ſuppoſe in the Figure the



Line ACB divided in C, and two Weights hung at the Arms CB and AC reciprocally proportional to the Length of thoſe Lines, then by what I prov'd under my laſt Head, thoſe Weights will mutually ſupport each other, and the Machine is called a Ballance. But if inſtead of the Weight at the End of the longeſt Arm, we ſhould ſuppoſe a Man to apply a Force equal to that of the Weight, then it is called a Lever. Now in this Caſe inſtead of ſaying the two Weights are reciprocal to each other, we ſay the Weight and the Power are reciprocal. So that the Effect is the ſame, but the Cauſe alter'd not in Quantity, but in Kind. From hence it follows, that the Mathematical Idea of a Lever of the firſt Kind, is an inflexible right Line A C B, one Point of which being ſupported by a Line C M at right Angles to it, the two Parts of the Lever impell'd by a Force or Forces, deſcribe Arches round that Point; which is the ſame as the Ballance. Therefore all that has been prov'd of the Ballance may be without farther Trouble apply'd to the Lever of the firſt Kind; the Line C M being called the Prop or Fulciment, and the Point C upon which the Lever reſts, the fixed Point.

THERE is however one thing to be obſerv'd in Practice, which produces an Effect in the inclined Lever, that is not found in the Ballance, which it is convenient to take ſome Notice of. In the Ballance the Weight generally acts upon the Arms, by pulling from below, by the means of one or more Strings faſten'd to it;

which meeting together in one Point at the Ballance, act only upon one Point of each Arm by the Nature of the Machine. But in the Uſe of a Lever, the Weight being generally faſtened to it, or elſe lying upon it, acts upon ſeveral Points of the Lever at once, and therefore when we are comparing the Diſtances of the Weight to that of the Power, we muſt conſider to what Point the Diſtance of the Weight muſt be computed. For ſince the Weight is in ſome Part of it nearer to the Center, as E F is

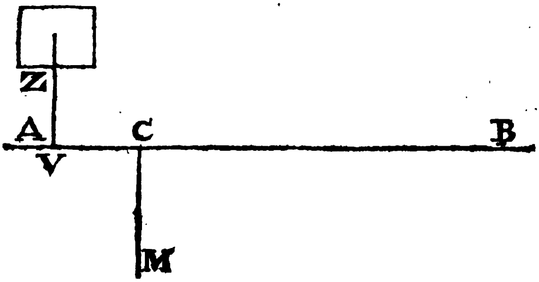


nearer than D A in the Fig. no Reaſon is aſſign'd why we ſhould take C F rather than C A for the Diſtance, which would make our Computation uncertain. Now in order to determine that Matter, the

Anſwer is, that this Diſtance muſt be taken to the Point V, where a Line let fall from the Center of Gravity of the Weight interſects the Lever. But this Term of the Center of Gravity of a Body, being new, it will be convenient to explain it a little.

THE Centre of Gravity of a Body is that Point round which all the other Parts would remain in Equilibrio, if that Point only were ſupported. In homogeneous Bodies the Center of Magnitude is the Center of Gravity. As for Example, if we ſuppoſe a Plate of Iron cut into a circular, ſquare, or triangular Form and a fine Needle ſhould be placed underneath it to ſupport it, in ſuch ſort that the Needle ſhould exactly hit the Center of this Circle, Square or Triangle, all the Parts of the Iron Plate will be ſuſpended and remain in Equilibrio round that Needle ſo apply'd. So a homogeneous Cube will remain in Equilibrio upon the Point of the ſame Needle, if it be ſo apply'd as to touch exactly the End of any Line which paſſes thro' its Center, and alſo to coincide with that Line. A few Tryals made upon proper Bodies, will ſhew much better than many Words, that there is in all Bodies ſuch a Centre, round which they will remain in Equilibrio, and that this is commonly in or near the Centre of Magnitude.

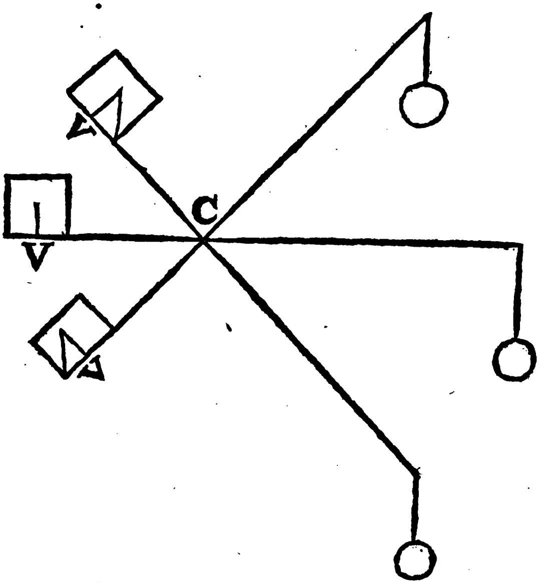
NOW I ſay, that this being admitted, if from the Centre of Gravity of the Cube A D E F, there be let fall a Perpendicular to V, the whole Cube A D E F preſſing upon the Line A F of the Lever, will have juſt the ſame Effect in turning the Lever round, as if it were to be hung by a String at the Point V, and by that Means act upon that Point of the Lever only. For if we ſuppoſe the Lever taken away, and inſtead thereof the Needle X V to be applied at V, by what I ſaid before, the Needle X V will ſupport the Cube. Therefore ſince the Needle at X V ſupports the Cube, it is preſs'd by the Cube with all its Weight; but the Line A F of the Lever is alſo preſs'd by the Cube with all its Weight; therefore that Line A F, and the Point V, are preſs'd with equal Preſſures; the Preſſure which is diſperſed all over the Line A F in the firſt Caſe, being here united upon the Point V. But if we ſhould imagine the Cube A D E F inſtead of being ſupported by the Needle X V, to be ſuſpended by a String at the Point V, it would ſtill have the ſame Effect, and the Effect of the Weight would be the ſame at V (X V being now ſuppoſed to repreſent a String) in that Caſe alſo, as before when it was diſperſed all over A F. By the ſame Reaſoning if we ſhould imagine in the Fig. a perpendicular inflexible Line V Z erected at V,

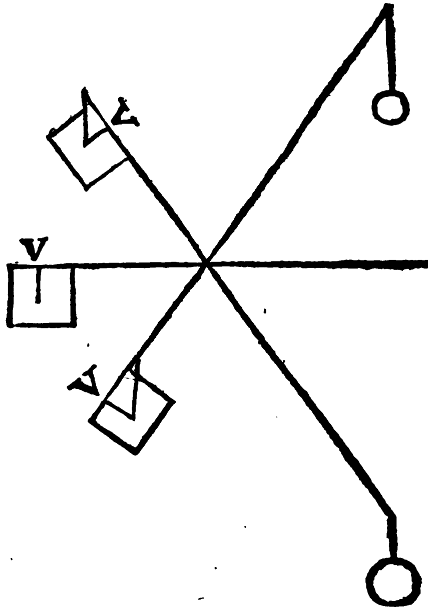


V, and the uppermoſt Point Z thereof to ſupport the Cube in like manner as the Needle X V did before, then it is plain, that the Cube preſſing upon this inflexible Line V Z, will by the Means thereof, communicate a Preſſure at the Point V, equal to the others alſo. From hence the following Concluſion naturally ariſes.

THAT if from the Centre of Gravity of any Body, there be drawn a perpendicular Line, that Body will have the ſame Effect in any one Point of that Line that it will have in any other. From thence may be deduced theſe Propoſitions.

THAT in a Lever inclined to the Horizon, as in this and the following Figure, if the Weight be faſten'd *upon* the Lever, it will require a leſs Force to ſuſtain it, when the Weight is elevated above the horizontal Line, than it does in a horizontal Poſition; and on the contrary if it be below the horizontal Line it will require a greater. For it was ſhewn, that the Diſtance of the Weight from the Prop,

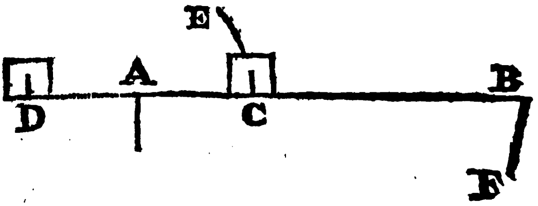




Prop, muſt be computed to that Point, where the Perpendicular from the Centre of Gravity of the Weight meets with the Lever.

BUT it is plain by inſpection, that in the former Caſe, this Perpendicular falls nearer to the Center C, than the Point V. Therefore by what was prov'd, as this Diſtance is leſs, ſo the Power requir'd to ſuſtain the Weight muſt be leſs. And in the latter Caſe it is equally plain, that the Diſtance is greater, and therefore the Power muſt be greater. But if inſtead of tying the Weight *upon* the Lever it were to be faſten'd *under* it, then by the ſame Way of Reaſoning the direct contrary would happen. This is ſo plain from the former, and by Inſpection of the Schemes, that for Brevities ſake I paſs it over; and only obſerve from both, that if there ſhould be Occaſion to ſupport a Weight by a Lever above the horizontal Line, it is moſt convenient to faſten it *upon* the Lever, and if in the contrary Poſition, *under* it. Theſe are the Properties of a Lever of the firſt Kind.

IN a Lever of the ſecond and third Kind there is but this thing to be demonſtrated, that if a Power ſupport a Weight, their Diſtances from the End, which is now the fixed Point, will be in reciprocal Proportion. If in the Fig. we ſuppoſe

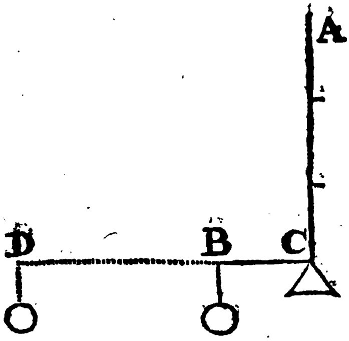


poſe the Line A C B to repreſent a Lever of this Kind, and that a Power at B ſupports a Weight at C, then I ſay, that the Power and the Weight are to each other in reciprocal Proportion of their Diſtances as before. Let us imagine the Lever prolong'd to D, ſo that A D be equal to A C. It was ſhewn before that equal Velocities cauſed in equal Bodies muſt be produc'd by equal Powers. But the Weight D equal to the Weight C, will by reaſon of its equal Diſtances from the Centre A, move when it is impell'd by a Force at B with the ſame Velocity as the Weight C. Therefore the ſame Power that can ſupport D will ſupport C alſo. But the Line D A B is a Lever of the firſt Kind, in which we prov'd, that the Power B was to the Weight D, as A D to A B. Therefore ſince the ſame

Power ſupports C equally, it will alſo be to the Weight C as A D to A B; or A D being equal to A C by the Conſtruction, as A C to A B. Q. E. D.

THE third Kind needs not a particular Demonſtration if we conſider rightly the Meaning of the Propoſition juſt now demonſtrated. The Meaning of which is, that if a Line ſhould have one End fix'd, and ſhould be capable by means of a Force or Forces apply'd to it to revolve round that Point as A; and that two contrary Forces are applied to it at C and B, the one at C impelling the Line downwards, and the other at B puſhing it upwards; (that is the Weight acting at C and the Power at B) that to make theſe oppoſite Forces counterpoize each other, the Quantity of each Force muſt be in reciprocal Proportion to its Diſtance from A. This is the Lever of the ſecond Kind. But if we ſhould ſuppoſe the Caſe inverted, and that at C the Force acts upwards and at B downwards, which is the Caſe of the Lever of the third Kind where the Power acts at C; and the Weight at B, yet it is I think very plain, that tho' the Direction of the Forces be alter'd, their Quantity muſt remain the ſame in order to counterpoize each other; that is, the Power will be to the Weight in reciprocal Proportion to their Diſtances from the fix'd Point or Fulciment A.

THE fourth Kind is where the fix'd Point or Fulciment is at an Angle, as



A C B in the Fig. In this Caſe alſo the reciprocal Proportion of the Diſtance ſtill holds, that is, the Power apply'd at A to ſupport the Weight at B, is to the Weight B as the Line B C to A C. Make

D C equal to A C, and let B C be to A C or D C, as 1 to 3. Now if we ſuppoſe the Weight at B to weigh three Pounds, the Force A which ſupports B = 3 Pounds at a Diſtance C B = 1, will ſupport one Pound D at a Diſtance C D equal to 3, as was before prov'd. But the Weight D is to the Weight B in reciprocal Proportion of the Diſtances B C and C D; whence it follows, that the Force at A at the Diſtance A C ſuſtaining the Weight D at the Diſtance C D, is alſo in the ſame Proportion; that is, the Power at A is to the Weight B, as B C is to C D or A C.

The Sum of what we have demonſtrated above, is contained in theſe Propoſitions.

1. That in all the four Kinds of Levers, when the Diſtances of the Weight and the Power are reciprocally proportional to the Quantities of their Forces; that is, when the Weight and Power multiply'd into their Diſtances produce

equal Rectangles on both Sides; the Weight will be barely ſuſtain'd by the Power. Whence it follows,

2. THAT if on the Side of the Power, that Rectangle be never ſo little the greater, the Power will prevail and move the Weight.

3. THAT the Diſtance from the Weight to the fix'd Point or Fulciment, is to be reckon'd from the Point of the Lever, which is interſected by a Perpendicular let fall from the Center of Gravity of the Weight, ſo that in an inclin'd Lever, the Power requiſite to ſupport it is in ſome Caſes leſs, and in others more, than is neceſſary to ſupport the ſame Weight upon an Horizontal Lever; in which the Lever differs from the Ballance, where the ſame Weights in all Poſitions whatſoever of the Ballance counterpoize each other, as was ſhewn.

4. THAT the leaſt imaginable Power may ſuſtain the greateſt imaginable

Weight, if the Length of the Lever be ſo divided, that the Diſtance of the Power from the Fulciment be as much greater than that of the Weight from the Fulciment, as the Weight is greater than the Power.

5. THAT if that Diſtance be made a little greater than in that Proportion, it will not only ſuſtain but move it. But that

6. THE Weight will move proportionably ſlower; the Velocities of both being, as I have often took Notice, proportionable to the Diſtances from the fix'd Point.

Of the PULLY.

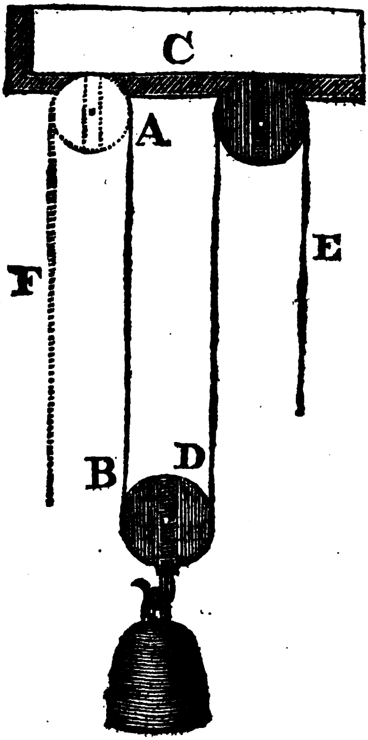
I Shew'd in my Diſcourſe upon the Lever, that the moſt that can be done by human Power in ſupporting of Weights that are ſo great as not to be managed by the natural Strength of a

Man, is to apply a Machine to thoſe Weights, by the Intervention of which, one may in ſuch a manner, proportion the Velocity with which the Power moves, to that with which the Weight moves, that they ſhall be to each other in reciprocal proportion to the Quantity of the Power and the Weight; that is, that the Part of the Machine apply'd to the Weight may move as much ſlower than that Part of the Machine, which is apply'd to the Power, as the Quantity of the Weight exceeds the Quantity of the Power. I then ſhew'd, how this may be done by the Application of a Lever, by which, if a Weight be ſupported by a Power leſs in Quantity than it ſelf, the Diſtances (and conſequently the Velocities) of each, muſt be in reciprocal Proportion to their Quantities.

BUT if we conſider the Nature of the Lever, we ſhall find, that as upon ſome Occaſions it will be of Service, becauſe, that by the Help of it one may apply a ſmall Power to the ſuſtaining or moving a great Weight, which is of great Uſe upon ſeveral Occaſions of Life; ſo on the other Hand for ſome other Uſes, it will be of no Service at all. For Example, if it be requir'd to raiſe a great and heavy Stone ſo high from the Ground, that it might be ſet upon a proper Carriage, in order to be tranſported to any Diſtance by the Help of that Carriage; in this Caſe the Lever will be of Service, becauſe that one or more of them being dextrouſly applied to the Stone, may lift it to ſo ſmall a Height as ſuch a Carriage generally needs to be of, perhaps 2 or 3 Foot. But if it were required to raiſe the ſame Stone 30 or 40 Foot high; in this Caſe the Lever will ſignify nothing. For if we ſuppoſe the Weight to be to the Power, only as 2 to 1, a Lever ſufficient to raiſe ſuch a Weight by the Help of ſuch a Power to ſuch a Height, muſt be above 90 Foot long, which is ſuch an Inſtrument as can neither be made nor manag'd in Practice. To ſupply this Defect therefore, the Pully was invented, which I now propoſe to ſpeak of.

THE Pully is a round Block of Wood, turning round a Pin, going thro' the Middle of it, in the Thickneſs of which there is cut a Groove of ſufficient Breadth and Depth, to admit a Rope to run in it with Freedom, and without rubbing againſt the Sides, at leaſt as little as is poſſible. This is ſo common a Machine, and ſo generally known by the Uſe it is of in a Jack, that I need ſay no more by way of Deſcription of it. In our Arguings concerning it, we are to ſuppoſe it perfectly circular, to turn with all poſſible Freedom about its Axis, which is ſuppoſed to have no Thickneſs, and the Rope made uſe of to be perfectly ſupple and pliant, and of no Thickneſs likewiſe. As to its Bigneſs or Diameter, we ſhould ſuppoſe it as ſmall as we can, becauſe, tho' in Practice ſuch Pullies will not be ſo convenient, by reaſon of ſome outward Accidents, yet ſince in theſe Speculations, we entirely ſet aſide the Conſideration of thoſe Accidents; it will be moſt convenient to imagine them ſo ſmall, as to bear no ſenſible Proportion to the Height of the Machine from the Ground, and alſo to have no Weight; that is, ſo that we may ſet aſide the Conſideration of their Bigneſs and Weight.

THE Manner of applying this Machine is as follows. The Height to which the Weight is to be raiſed being determined, there muſt be ſet up at that Height ſome firm Support, able to bear the whole Streſs both of the Weight and of the Power acting upon it at once. In Practice they make Uſe of the Sheers for this Purpoſe; or a ſtrong croſs Beam well ſupported on both Sides, when they would raiſe the Weight to greater Heights than the Sheers can be conveniently applied to. Then to this Beam is faſten'd a Pulley by the Help of ſome Contrivance, which at the ſame time that it keeps it tight, and cloſe, and firm to the Beam, gives free Leave to the Pully to play about its Axis. Then into the Groove, that goes round the Thickneſs of the Pulley, they ſlide the Rope, which naturally hangs down looſe on both Sides of the Pully, and one End of the Rope is taken hold of by the Perſon that is to draw up the Weight. At the ſame time there is prepar'd another Pully below, which has a Hook, or other proper Device to take hold of the Weight with, faſten'd to it in ſuch manner, as that the Pully may play freely about its Axis, while the Hook remains unmoved and ſtedfaſt, by a like Contrivance with that of the upper Pully. Then the other End of the Rope that hangs looſe being let down, is taken hold of by thoſe below, and put round the lower Pully; from whence it is again carried up and faſten'd at the Top by ſome Device to the croſs Beam. This Faſtening is made ſure and tight, and as ſtrong as they can poſſibly make it, becauſe upon that Part of the Beam the Force and Streſs of the whole Machine depends. Then will the whole appear as in



in the Fig. This being done, the Man that is to raiſe the Weight, pulls his End of the Rope with all his Force downwards, which increaſing the Length of the Rope towards him, muſt needs ſhorten it every where elſe. Therefore the Length of the Parts of the Rope from the fix'd Point to the lower Pully, and from the upper Pully to the lower Pully being ſhorten'd, it follows, that the lower Pully muſt riſe, the Diſtance between it and the croſs Beam at top becoming ſo much leſs as thoſe two Parts of the Rope are made ſhorter. Then giving a ſecond Pull, he raiſes it as much higher, and ſo on till at laſt it arrives at the top, carrying with it the Weight faſten'd to it by the Hook.

FROM this Deſcription, one may, without much Trouble, deduce the ſeveral Properties of the Pully, the firſt of which is, that if the Power be equal only to half the Weight, it will ſupport it by the Help of this Machine.

TO prove this, I muſt premiſe by Way of Preparation, an Axiom or Law of Nature, relating to the Communication of Motion, which I have not yet taken Notice of, which is, THAT in Bodies acting upon each other, the Body acted upon doth communicate to that which acts upon it a Re-action, which is contrary and equal to the Action of the other Body upon it ſelf; that is, to expreſs my ſelf more univerſally and conciſely, that Action and Re-action are contrary and equal to each other.

To explain this a little. If a Man preſſes with his Finger upon a Stone, the Stone by Re-action preſſes equally upon his Finger. If a Man is driving a Nail into a Piece of Timber with a Hammer, the Nail acts upon the Hammer by driving it back, as much as the Hammer acts upon the Nail in driving it forwards. If a Man be pulling a Stone towards him by means of a Rope faſten'd to it, he is by the Re-action, pull'd as much backwards towards the Stone, as the Stone is pull'd forwards towards *him*. If a Weight be ſuſpended by a Rope from a Beam, by that Means it acts upon that Part of the Beam it is ſuſpended from, pulling it downwards, with a Force equal to its Weight. In this Caſe, the Beam ſhall by its Re-action pull the Weight upwards from above, exactly as much as it ſelf is pulled downwards by the Weight from below.

To apply this. The Weight hanging from the lower Pully in the Machine before deſcribed acts upon the two Strings on each Side, each of which, for that Reaſon muſt ſuſtain half the Weight only. Therefore the Weight tends to pull Part of the String D C E over the Pully at C, with a Force equal to that of half its Weight; but we ſuppoſed the Power pulling at E in a contrary Direction to be equal to half the Weight; therefore theſe two Forces being equal, will ſuſtain each other mutually. The other half of the Weight acts upon the String A B, pulling it downwards, but by the Law of Re-action it will be pull'd upwards by an equal Force; therefore on that Side alſo, it will be kept ſuſpended by a Force equal to that with which it endeavours to deſcend. Hence it follows,

1. THAT if we ſuppoſe the Machine to be alter'd, and the fix'd Point A being remov'd, a ſecond upper Pully to be be placed in its room; and that the String B A were ſo lengthen'd, as that it might be thrown over this ſecond upper Pully and hang down to F; and that a new Force equal to half the Weight ſhould be apply'd at F pulling the String A F downwards. I ſay, that this Force would be equal to that, by which the fix'd Point A ſuſtains its Share of the Weight. For the Force by which the fix'd Point A ſuſtains the Weight, is equal by the Law of Action and Re-action, to that with which the Weight acts upon *it*; but by what was ſaid before, the Weight acts upon it with half its Force; therefore the Force of the Point A in ſuſtaining the Weight is equal to half the Weight. But this new Force acting by this new Pully is equal to half the Weight, therefore it is equal to the Force of the

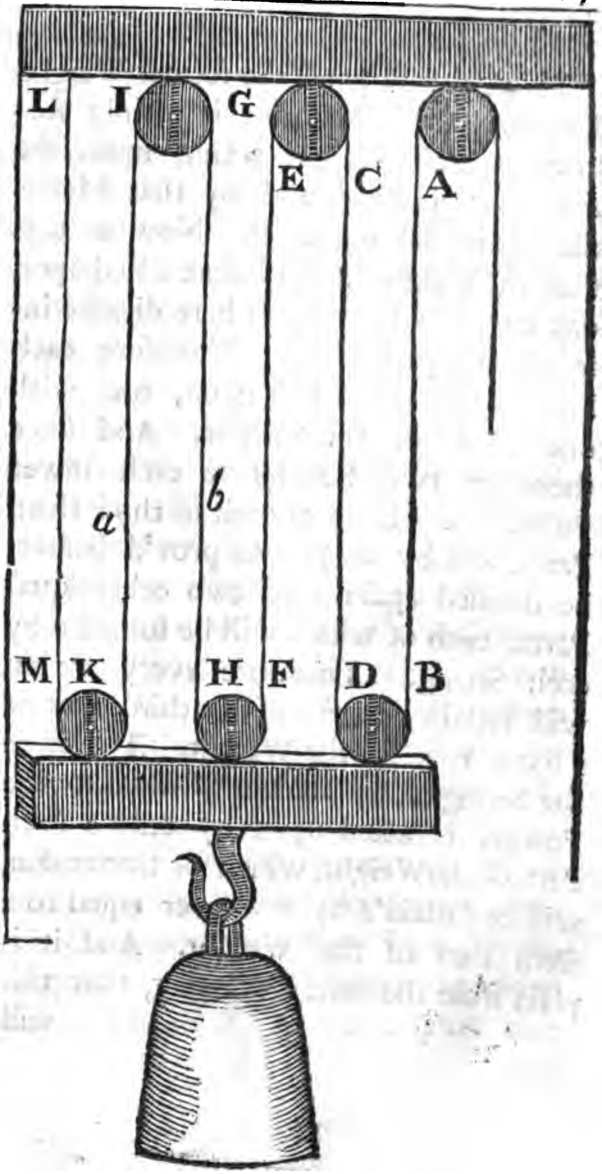
Point A. From hence we conclude, ſuppoſing this new Pully to be now remov'd,

2. THAT the String A B, and the String C D, are equally ſtrain'd by the Actions of the Weight, and Power, and fix'd Point upon them. For the Weight ſtrains each of them with a Force equal to half its Weight; and the Power, and fix'd Point on their Side, ſtrain them equally, with Forces ſeverally equal to half the Weight; therefore being ſtrained by equal Forces, they muſt be equally ſtrained.

3. THE Power will be to the Weight, as Unity to the Number of Strings apply'd to the lower Pullies; that is, in this Caſe as 1 to 2; the Number of Strings apply'd to a ſingle lower Pully being two, as is plain by Inſpection. But the Power was ſhewn to be to the Weight as 1 to 2; therefore as Unity to the Number of Strings.

4. If the Number of upper and lower Pullies ſhould be increaſed, theſe Propoſitions will remain true in like Manner as before.

The moſt ſimple Way of conceiving this done, is by ſuppoſing the lower Pullies to be faſten'd upon a croſs Piece, and the Weight to be hung exactly in the Centre of Gravity of this croſs Piece and of the Pullies taken together, by a Hook, as in the following Figure.


L
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If we then ſuppoſe the former Weight hung upon this Machine, it plainly follows, that the Weight acting upon the croſs Piece pulls it, and by that Means the Pullies faſten'd to it. Now in this Caſe the Weight which before acted upon one lower Pully only, is here divided into three equal Parts. Therefore each lower Pully is acted upon, but with one third of the Weight. And ſince there are two Strings to each lower Pully, the Action of each of thoſe third Parts, will by what was prov'd before, be divided again into two other equal Parts, each of which will be ſuſtain'd by each String. Therefore every String will ſuſtain one half of one third, that is, a ſixth Part of the Weight. Therefore the String A B, which is next to the Power, is acted upon by only a ſixth Part of the Weight, which for that reaſon, will be ſuſtain'd by a Power equal to a ſixth Part of the Weight. And it is plain from the ſame Reaſoning, that this will be the Caſe, let the Number of Pullies be never ſo great.

AND the Reaſon of this will appear, if we conſider ſomething nearer the Make of this Machine. For by the Nature of it, the Weight acting upon the String K I, tends to pull that String downwards, from the Pully between I and G, and by that Means to make the ſeveral Parts of it to deſcend towards the Point K. But if any Point thereof, as *a* deſcends, it muſt make the Point *b* in the String G H to aſcend. But by the Nature of the Machine, all the Strings are pull'd downwards equally, and therefore the Point *b* tends to deſcend with the ſame Force as *a* does; for which Reaſon, neither can one deſcend nor the other aſcend. Therefore thoſe two Parts of the Weight, that are ſupported by the Strings I K and G H, are not at all ſuſtain'd by the Power, but by their mutual Counterpoize; that Part of the Weight that acts at K, being ſupported by the String G H, and that Part that acts at H by the String I K, acting with equal Forces contrary Ways. In this Caſe, the horizontal Diameter of the Pully I G may be conſider'd as a Ballance or Lever, in which the two Arms are equally diſtant from the Centre, and two equal Forces pull thoſe two equal Arms downwards, in which Caſe it was ſhewn before, that theſe two Forces would mutually ſupport each other.

AND by the ſame Manner of arguing, one might ſhew, that the two Parts of the Weight acting upon E F and C D, ſupport each other mutually. Now here being four Parts of the Weight out of the ſix, which as we have ſhewn, ſupport each other, it follows, that we have but two to provide Forces for. But the Part of the Weight applied to L M, is ſupported by the fix'd Point at L, with a Force equal to that with which it acts upon L, by the Law of Action and Re-action mention'd before. So that all the Weight the Power has to ſupport by its own Strength, is that ſingle Part only that acts upon A B, which being a ſixth Part of the Weight, it is plain, that a Power equal to a ſixth Part of the Weight, will be ſufficient for that Purpoſe as was ſhewn before.

I have dwelt the longer upon this, becauſe there is ſomewhat particularly ſubtle and worthy Obſervation in the Contrivance of this Machine, wherein ſo great a Part of the Weight ſupported by it is thrown off from the Power, and made as it were to ſupport it ſelf.

FROM hence it follows, as before,

1. THAT the Strings are equally ſtrain'd, being pull'd by equal Forces. For in this Caſe we ſhew'd that each String is ſtrain'd by a ſixth Part of the Weight, therefore equally.

2. THAT the Power is to the Weight, as Unity to the Number of Strings applied to the lower Pullies; for as appears by Inſpection, the Number of thoſe

Strings is 6, and it was ſhewn, that the Power is to the Weight as one to ſix, And that this will be the Caſe when the Number of Pullies is greater, there can be no Pretence of Reaſon to doubt,

3. SINCE the Power that only *ſupports* a Weight by the Help of Pullies; muſt be to the Weight as Unity to the Number of Strings applied to the lower Pullies, it follows, that a Power that *moves* a Weight by the Help of Pullies, muſt be in a greater Proportion to the Weight, than that of Unity to the Number of Strings; becauſe a Power that *moves*, muſt be greater than that which barely *ſupports*, And by how much the Quantity of the Power exceeds this Proportion, with ſo much the greater Velocity it will cauſe the Weight to aſcend towards the fix'd Point.

4. THAT in this Aſcent the ſeveral Parts of the Strings will move with different Velocities, that are as the Lengths of the String taken between any of thoſe

Parts and the fixed Point L. For the entire Cord may be conſider'd as a flexible Lever of the ſecond kind, in which the fixed Point, or Fulciment, is at one End L, and the Power acts at the other End at the Pully A; in which Caſe, by what was ſhewn before, the Velocities of the Parts of the Lever will be as their Diſtances from the Fulciment. If we ſuppoſe the lower Pullies raiſed almoſt as high as the upper Pullies, this Reſemblance between the Cord and the Lever will appear more plainly.

5. THAT by this Inſtrument alſo one may move the greateſt imaginable Weight, with the leaſt imaginable Power, by increaſing the Number of Pullies, ſo as that the Number of Strings apply'd to the lower Pullies, may be to Unity, as the Weight to the Power.

Of the WHEEL *and* AXLE, *ſometimes called the* Axis in Peritrochio.

THE Wheel and Axle has all the Advantages belonging to the Pully, with ſome peculiar to it ſelf. It may not unfitly be divided into two Kinds, ſimple and compound.

THE Simple conſiſts of two Cylinders, having one common Axis, but unequal Baſes. That with the larger Baſe is called the Wheel, and the other the Axle. So in Fig. 1. the Cylinder whoſe Baſe is A B, is called the Wheel, and the Cylinder whoſe Baſe is C D, is called the Axle. In Practice, their Baſes and Altitudes are generally ſo proportioned to each other, that the Altitude of the Wheel is leſs than the Diameter of its Baſe; and on the contrary, the Altitude of the Axle exceeds the Diameter of its Baſe.

THE Manner of uſing this Machine is as follows. The two Ends of the Axle, Pag. 104.

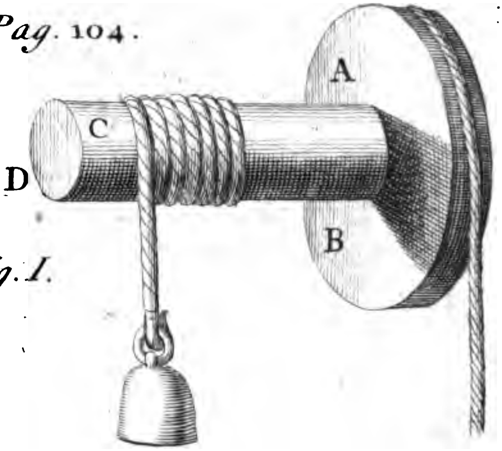


Fig. I.

Pag. 110.

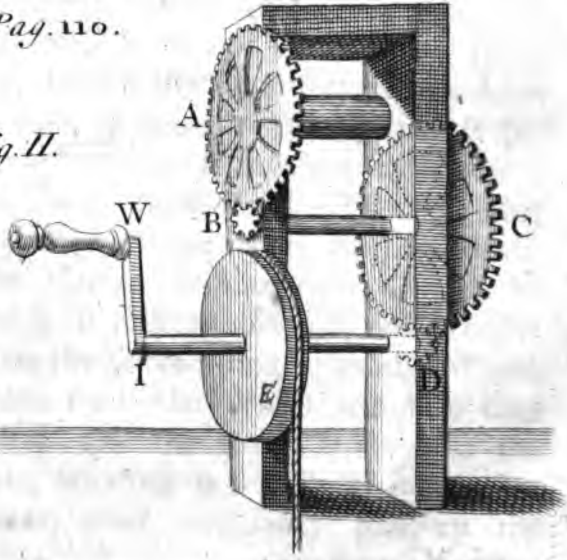


Fig. II.

Axle being ſo ſupported in an horizontal Situation, that the whole Machine may freely revolve about the common Axis of the Wheel and Axle; the Wheel by ſome outward Force applied to it, is made to turn round, which obliges the Axle to turn round likewiſe. This is ſometimes done by Means of a Rope going round the Wheel, and faſten'd to ſome Place in its Circumference; ſometimes by Men walking within it, as in the ordinary Cranes uſed for lifting and removing Goods; which are only Applications of this Power. Another Rope is alſo faſten'd about the Axle, in ſuch a Manner as to wind it ſelf round it, while it revolves by means of the Force applied to the Wheel. That End of the Rope which hangs down is faſten'd to an Hook or other Device, by which it takes hold of the Weight. Then the Force acting upon the Wheel, makes both the Wheel and Axle turn round; and the Rope faſten'd to the Axle, winding it ſelf about it by that means, muſt neceſſarily pull up the

Weight; the Portion of Rope between the Machine and the Weight, continually growing ſhorter and ſhorter, by Reaſon of its winding it ſelf about the Axle as faſt as the Wheel turns round. Therefore the Weight muſt aſcend continually towards the Machine, till ſuch time as the whole Rope being wound about the Axle, it is made to riſe as high as the Force that acts upon it.

FROM this Deſcription of the Machine and its Uſe, we may obſerve, Firſt, that here are two contrary Forces acting in Oppoſition to each other, one whereof, that is the Weight, tends to make the Machine revolve one way, and the other, that is the Power, tends to make it revolve another way. Secondly, that theſe Forces act at different Diſtances from the Axis, or immoveable Line, round which the Machine turns; namely, the Weight at a Diſtance equal to the Radius of the Axle; and the Power at a Diſtance equal to the Radius of the Wheel. Now this anſwers to a

Lever of the ſecond Kind, where the fixed Point is at the End of the Lever, and the Weight between the Power and the fixed Point. Therefore what was diſcovered and ſhewn under that Head relating to ſuch a Lever, may be alſo applied to this Machine, expreſſing the Diſtance of the Weight by the Radius of the Axle, and the Diſtance of the Power by the Radius of the Wheel.

NOW it was there found, that where the Diſtance of the Weight and the Power from the fix'd Point are reciprocally proportional to their ſeveral Quantities, the Weight would be ſuſtained by the Power, and from thence it was concluded, that if either the Quantity, or the Diſtance of the Power exceed that Proportion, it would be ſufficient, not only to ſuſtain, but to move it. Hence it follows,

1. THAT if in this Machine the Radius of the Axle be as much leſs than the Radius of the Wheel, as the Weight

is greater than the Power, the Power will ſupport the Weight; that is, if the Radius of the Axle be as one, and the Radius of the Wheel as three; and if the Weight be as three, and the Power as one, that then the Power will *ſupport* the Weight. And therefore, that if either the Quantity of the Power, or the Radius of the Wheel be greater than in this Proportion, the Power will *move* the Weight. It was alſo inferr'd, that the leaſt imaginable Power, might ſuſtain the greateſt imaginable Weight, if the Diſtance of the Weight from the fix'd Point, were as much leſs than the Diſtance of the Power from that Point, as the Power is leſs than the Weight; and if that Diſtance of the Weight be ſtill leſs, it would move it. Hence we conclude, 2. That in the Wheel and Axle, if the Radius of the Wheel exceed the Radius of the Axle, as much as the Weight exceeds the Power, that with this Machine alſo, the leaſt imaginable Power may ſuſtain the greateſt imaginable Weight; or if the

Exceſs be ſtill greater in than that Proportion, it will alſo move it.

FROM what has been ſaid, it appears, that by increaſing the Radius of the Wheel, in Proportion to the Radius of the Axle, one may by the Help of this ſimple Machine alone, remove as great Weights as uſually come in the way in common Life; only by diminiſhing the Radius of the Axle, or increaſing that of the Wheel, in ſuch a Manner, that theſe Radii may bear the ſame Proportion to each other reciprocally, as the Power and Weight do.

BUT when we come to apply this Speculation to Practice, it will be found, that if the Weight exceed the Power very much, one of theſe phyſical Hindrances will happen. Either the Axle muſt be made ſo ſlender as not to be able to ſupport the Weight, or the Radius of the Wheel muſt be ſo large, as many times it will be impoſſible to make or manage ſuch a Machine as would be neceſſary.

Now to ſupply this Defect, the compound Wheel (if I may ſo call it) was invented. The Contrivance is after this Manner. The Axle being made of ſufficient Strength and Thickneſs to ſupport the Weight, the Wheel belonging to it is made of ſuch a Radius as they find moſt convenient, and the Circumference of the Wheel is cut into Teeth as in the Fig. Then either above or below (it matters not which) there is placed a leſſer Wheel cut into Teeth of equal Bigneſs, or rather a ſmall matter leſs than thoſe in the large Wheel. This leſſer Wheel is called the Pinion. This Pinion is alſo faſten'd to another Axle, and ſo placed, that the Teeth of the Wheel and Pinion may take hold of each other, as they both turn round. At the End of this ſecond Axle is faſtened another large Wheel, whoſe Circumference is divided into Teeth like the former; and under it is placed a ſecond Pinion faſten'd to another Axle. At the other End is faſten'd another large Wheel, to which the Force that turns the whole Machine is to be applied. All this is ſhewn in the Figure. One may if there be Occaſion, multiply the Number of the Pinions and Wheels ſtill farther; but as the Method is exactly the ſame, it will be to no purpoſe to ſpend Words in carrying on the Deſcription farther. Now by this Means the ſame Effect is produced, as if the Radius of the firſt Wheel had been increaſed, and it is very plain, it is done much more eaſily and conveniently. How this comes to paſs, I ſhall now proceed to explain.

SUPPOSE it were required with a Power equal to 40 lib. to ſuſtain a Weight of [illegible] Pounds, which is to the Power as 108 to 1. Suppoſe alſo, that an Axle made of proper Materials, of 4 Inches Radius, that is, of 8 Inches Diameter, were juſt able to ſupport ſuch a Weight without breaking. Now if we make uſe of the ſimple Machine to ſuſtain this given Weight, within this given Power, we ſhall find that it will require to an Axle of 8 Inches Diameter, a Wheel of 72 Foot in Diameter to ſuſtain the Weight. But ſuch a Machine is impoſſible to be made, or to be managed by ſuch a Power. Therefore the compound Wheel muſt be uſed.

To demonſtrate its Effect, I muſt premiſe, what I prov'd before in general, and confirm'd by the Examples of thoſe mechanical Powers I have treated of hitherto, *viz.* That where the Quantity of a Power, and the Velocity with which it moves, multiply'd into each other produce a Rectangle or Product equal to that of a Weight multiply'd to its Velocity, the Power and the Weight will be in Equilibrio. That is, if a Power be = 1, and its Velocity = 20; and a Weight be = 20, and its Velocity = 1, this Power and Weight will mutually ſupport each other; the Rectangle on each Side being = 20. Therefore if I prove, that the Velocity of the Power may by the Help of this compound Wheel be ſo increaſed, as when multiply'd into the Quantity thereof, of, to be equal to the Velocity of the Weight multiply'd into its Quantity, I ſhall have prov'd at the ſame time, that this Power apply'd to this Machine will ſuſtain the Weight.

LET the uppermoſt Wheel be called A, the Pinion under it B, the ſecond Wheel C, the ſecond Pinion D, and the third Wheel E. Let the Wheel A be 12 Inches Radius, that is, 2 Foot Diameter, and let the Axle be, as was ſaid before, 4 Inches Radius. Imagine the Power apply'd at the Circumference of the Wheel A, and by what was ſhew'd before, its Velocity will be to that of the Weight, as the Radius of the Wheel to the Radius of the Axle, that is, as 12 to 4, that is, as 3 to 1. Let us now imagine the Radius of the Pinion B under it to be two Inches, and the Number of its Teeth = 12, and thoſe of the Wheel A = 72, that is, as their ſeveral Radii, that is as 2 to 12, that is as 1 to 6. Let the Wheel C be equal in Radius, and in Number of Teeth to the Wheel A. We are now to ſuppoſe the Power applied at the Circumference of this Wheel C, and to find its Velocity there. Now the Weight being ſuppoſed to act upon the Axle of the Wheel A, makes the Circumference of that Wheel revolve with a Velocity that is to its own as 3 to 1. As its Teeth go round they catch ſucceſſively upon thoſe of the Pinion B, and make that Pinion go round with a Velocity equal to their own; that is, three times faſter than the Weight; for the Teeth catching upon each other, can move neither ſwifter nor ſlower than each other. The Pinion B in going round, makes its Axle and the Wheel C go round likewiſe. But the Radius of the Wheel C being to that of the Pinion B, as ſix to one, the Velocity of its Circumference, and therefore of the Power ſuppoſed to be apply'd there, will be to that of the Pinion B, as ſix to one alſo. But the Pinion B is three times ſwifter than the Weight, therefore the Velocity of the Power at the Wheel C, being 6 times greater than that, muſt be 18 times greater than that of the Weight. Suppoſe, laſtly, the Power apply'd at the Circumference of the Wheel E. This Wheel having no Pinion to take hold of, has no Teeth, but is made like that in the ſimple Machine with a Rope for the Power to move it by. Let the Pinion D be equal in Radius, and in Number of Teeth to the Pinion B, and let the Radius of the Wheel E be equal to that of the Wheels C and A. Then by what was ſhewn before, the Teeth of the Wheel C, taking hold on thoſe of the Pinion D, make that Pinion turn round with a Velocity equal to their own. That Pinion turning round, makes the Wheel E turn round alſo with a Velocity, that is at the Circumference, as the Radius of the Wheel to the Radius of the Pinion, that is as ſix to one. But the Velocity of the Pinion D is 18 times greater than that of the Weight, as was ſhewn before; therefore the Velocity of the Power at the Circumference of the Wheel E, being ſix times greater than that, will be to that of the Weight, as

6 × 18 to 1, that is as 108 to 1. Therefore the Power, whoſe Quantity is as one has a Velocity as 108; and the Weight whoſe Quantity is as 108 has a Velocity only as one. Therefore this Power, and this Weight multiply'd ſeverally into their Quantities and Velocities, produce equal Rectangles; therefore they will mutually ſupport each other. Q. E. D.

FROM hence it is eaſy to obſerve, 1. The vaſt Convenience of this compound Machine, which within a ſmall Compaſs, performs the ſame Thing as would have required a ſingle Wheel of 72 Foot Diameter.

2. THE vaſt Advantage it has over the Pully in the ſmall Space taken up by the Machine; the ſame Thing being done with three Wheels and two Pinions, as would have required 108 Pullies, upper and lower.

3. THAT the Velocity of the Weight is to that of the Power in a Proportion compounded of the Proportion of the Radius of the uppermoſt Axle, to that of the Wheel A, into that of the Pinion B to the Wheel C, multiply'd into that of the Pinion D to the Wheel E. This laſt Corollary is of great Uſe, for all the Contrivance of Clocks and Watches depends thereon; but the Importance of the Subject requires that it ſhould have a particular Conſideration; too long to be entered upon here.

THOSE who are deſirous to be farther informed concerning the Application of this Corollary to thoſe Machines, may find Satisfaction in a little Treatiſe, called, *The Artificial Clock-maker*, publiſhed ſome time ago.

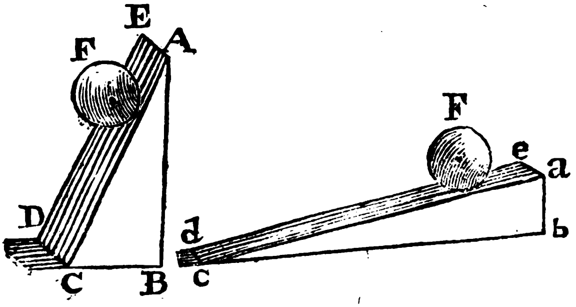
SOMETIMES in Practice, they find it more convenient to join a Winch, as W I, inſtead of a Wheel, to that Axle which impreſſes the Motion on all the reſt, as in the Figure. If that Length W I be equal to the Radius of the Wheel E, its Effect will be the very ſame, as that of the Wheel; if greater or leſs than that Radius, its Effect will be proportionably greater or leſs; it being the very ſame Machine under a different Form.

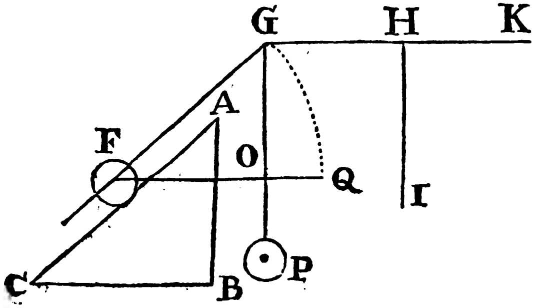
Of the INCLINED PLANE.

SOMETIMES the Weight that is to be moved is of the Figure of a Sphere or Cylinder, or elſe can be conveniently fixed to ſome proper Machine, by the Help of which, it may roll forwards upon a Plane, when that Machine is impelled by an outward Force. In that Caſe to raiſe the Weight to a given Height, the inclined Plane may be made uſe of. The Meaning of this hard Word amounts in Practice to no more than this; that if a Man has occaſion to ſet a Weight upon an Eminence, and the

Weight be ſo great, that he can't lift it by his natural Strength, he will take a long ſtout Plank, or ſomething equivalent thereto, and ſetting it ſloping, puſh the Weight before him up the Plank to the Place he deſigns to ſet it in; and ſuch a Plank or other Contrivance like it, is what the Mathematicians call an inclined Plane. Now 'tis evident, that the ſhorter this inclined Plane is, the ſteeper is the Aſcent; and the longer the Plane is, the Aſcent muſt be the eaſier. It is plain alſo, that it is much eaſier to puſh a rolling Weight up a Hill that riſes gently, than up a Hill that is very ſteep. This is univerſally known in general; but the exact Proportion of the Weight to the Power at any Degree of Inclination or Slope, and the moſt convenient Manner of applying the Power thereto, are Matters of Geometrical Speculation. We muſt ſuppoſe in all Caſes, that the inclined Plane is perfectly even, ſmooth and hard, ſo that the Weight may not ſink into it, nor be diſturbed in its Motion upwards by any outward Cauſe.

The firſt Thing to be found is, what thoſe different Forces are, with which the ſame Weight will deſcend at different Degrees of Inclination. Let the Weight F be rolling down the Plane E A D C





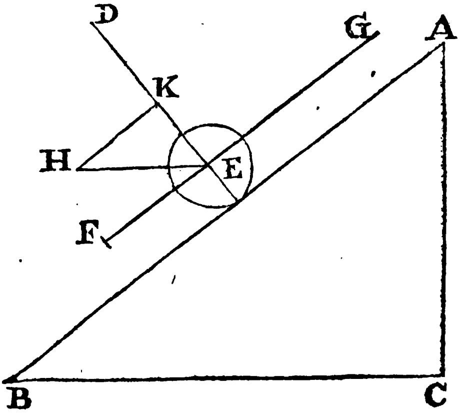
in the Figure, it is plain, that it will require a greater Force to ſuſtain it, than it would do, if the ſame Weight were rolling down the Plane e a d c. In order then to find exactly the Proportion of theſe Forces, let us ſuppoſe the right angled Triangle A C B to repreſent the Priſm E A D C B, and let the Hypothenuſe A C repreſent the Plane E A D C of that Priſm. Let the Weight F be rolling down the Side A C; and from the Centre of the Weight, let there paſs a String F G parallel to A C, faſtened to the Arm G H of the Ballance G H I K. Then the Force with which the Weight, by Means of this String pulls that Arm of the Ballance, will be equal to the Force with which it tends to deſcend along the Line A C. Let the ſame, or an equal Weight be ſuppoſed to hang at P in a perpendicular Situation. Then by what was ſhew'd, *p.* 28, 29, the Force with which the Arm G H is pulled obliquely from F by the Weight, is to the Force, with which the ſame Arm is pulled by the ſame Weight perpendicularly from P, as the Line G O is to

F G, that is, as the Sine of the Angle G F O is to Radius. But the Angle G F O is equal to the Angle A C B, the Angle of the Inclination of the Plane to the Horizon. Therefore the Force, with which the Weight pulls the Ballance from F; or which is the ſame thing with which it tends to deſcend from A to C, will be to its whole Weight, as the Sine of the Angle of the Inclination of the Plane to the Horizon, is to Radius. But the Force, with which it tends to deſcend, being equal to a Force able to ſuſtain it, therefore it will follow alſo,

THAT a Force, which ſupports a Weight upon an inclined Plane, will be to the Weight, as the Sine of the Angle of Inclination of the Plane, is to Radius.

BUT this is upon the Suppoſition that the Power is applied in the moſt advantageous Manner; that is, in a Line parallel to C A. But there being other Manners of applying the Power, it will be proper in the ſecond Place to conſider their different Effects. For tho' every Man, no doubt, would chuſe to apply a Power in the moſt advantageous Manner, and therefore it ſeems to little Purpoſe to enquire into the Nature of thoſe other Applications; yet, becauſe it is neceſſary to omit nothing that relates to the Subject, they muſt be conſider'd in their Turn.

To this Purpoſe, ſuppoſe a Power applied along the Line of Direction D E



in the Figure, to the Weight at E, perpendicular to the Line A B. I ſay, that a

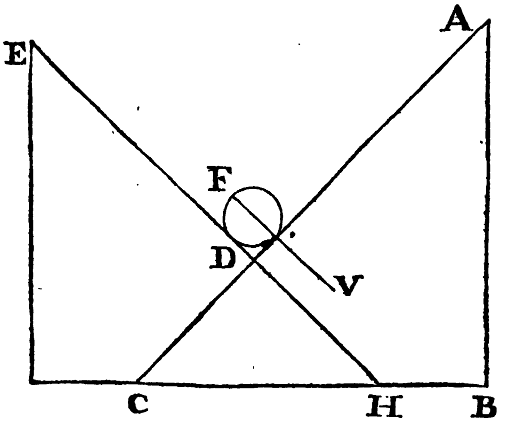
Power ſo applied, would have no Effect at all upon the Weight. For the Weight deſcending from A to C, moves in a Direction parallel to the Line A B, deſcribing with its Centre the Line F G. If the Power was applied therefore from F to G, it would have the greateſt Effect poſſible, either to ſuſtain it, or to retard its Motion; becauſe as has been ſhewn before, a Force applied to a Body in the Line of Direction in which it moves, will have the greateſt Effect poſſible upon it. And on the contrary, if the ſame Power were applied, the contrary Way from G to F, it would accelerate its Motion downwards the moſt that is poſſible. Therefore ſince the Power applied in one Direction drives the Weight up, and applied in the other Direction, drives it down; it follows, that in the intermediate Direction, which is D E, it will neither drive it up nor down; that is, it will have no Effect at all.

Hence we may determine the Effect of an oblique Force as H E, which is neither parallel to the Plane as F G, nor perpendicular to it as D E.

THE Force applied in the Direction H E, being ſuppoſed equal to the Force applied at F in the Direction F G, let the Line H E be made equal to F E, Draw H K perpendicular to D E, cutting D E in K. Then will the Force H E be reſolved into two Forces, H K and K E. The Force K E being perpendicular to the Plane, or which is the ſame thing to F G, will by what was juſt now ſhewn, have no Effect at all, The Force H K being parallel to the Plane, or to F G, ſupports the Weight; and by what has been ſhewn before, the Force H K is to the Force H E, as the Line H K is to the Line H E or F E. But H K is to H E, as the Sine Complement of the Angle K H E, or which is the ſame, of the Angle H E F, to Radius, Now the Angle H E F is the Angle in which the Line of Direction of the oblique Force H E is inclined to the Plane, or to F G, That Angle is called the Angle of Application. Therefore the Effect of the Force applied in the Direction H E, that is, whoſe Angle of Application is H E F, will be to the Effect of the ſame Force in the Direction F G parallel to the Plane, as the Sine Complement of the Angle of Application to Radius. And ſo in general, the Effects of ſuch oblique Forces will be as the Sine Complement of the Angle of Application.

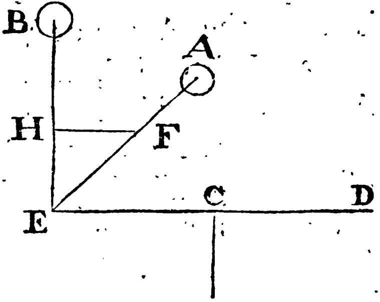
ANOTHER Thing convenient to be known relating to the inclined Plane, is the Quantity of the Preſſure of the Weight upon the Plane. For tho' we ſay, that the Weight is ſuſtained by the Power, yet that is not to be underſtood ſtrictly, as if the whole Weight preſs'd upon the Power, but only that the Power keeps the Weight from rolling down the Plane. The entire Weight is ſuſtained, partly by the Power, and partly by the Plane; and ſo much Weight as is ſuſtain'd by the Plane, the Power is eaſed of. The leſs the Angle of Inclination of the Plane is, the leſs Weight is ſuſtained by the Power, and the more by the Plane. And ſo on the contrary. What we have now to do, therefore, is to find a general Rule for knowing what Part of the Weight is ſupported by each.

SUPPOSE then a Plane D E at right Angles to A C to be ſet before the Weight, and to keep it from rolling downwards. The Preſſure of the Weight upon the Plane D E, will be as the Force with which it tends to deſcend. Now



that Force was ſhewn above to be to that of the whole Weight, as the Sine of the Angle of Inclination of the Plane A C to Radius. But ſince the Plane D E ſuſtains only a Preſſure of that Part of the Weight which is in that Proportion to the whole; it follows, that the Remainder muſt be ſuſtained by the other Plane A C or A D. Therefore ſince the whole Weight is as Radius, and that Part of it which preſſes upon the Plane D E, as the Sine of the Angle of Inclination of the Plane A C, it follows, that the Remainder of the Weight preſſing upon the Plane D A, will be as the Sine of the Compliment to 90° of that Angle. For if the Plane D A were taken away, and the Weight were to be ſuſtained by a Power applied in the Direction V F parallel to D E, that Power would be as the Preſſure of the Weight upon D A. But that Power would be, by what was ſhewn before, as the Sine of the Angle E H C, which is equal to the Complement to 90° of the Angle of Inclination D C B or A C B. But the Preſſure being as that Power, will be alſo, as the Sine of the Complement to 90° of the Angle of Inclination.

IT may not be amiſs, from what has been ſaid, to deduce the Proportion of the different Impulſes made upon Surfaces at any given Degree of Inclination, by moving Bodies impinging on thoſe Surfaces. In the Fig. let two equal Bodies B and A move along the right Lines BE and AE with equal Velocities, and let it be required to aſſign the different Quantity of Percuſſion of each of them upon the Arm of the Ballance E C D. From any Point of A E as F, let fall a Perpendicular to H; then will the Force A E be divided into two, F H and H E;



H E; whereof F H being parallel to E C D has no Effect at all, but H E being perpendicular thereto, has the ſame Effect as the oblique Force F E. Therefore, by what has been ſhewn before, as the Line H E to the Line F E, ſo is the Effect of the oblique Force to that of the perpendicular Force. But moreover as H E to F E, ſo is the Sine of the Angle H F E to Radius. But the Angle H F E is equal to the Angle F E C, which is called the Angle of Incidence. Therefore, as H E to F E, ſo is the Sine of the Angle of Incidence to Radius. But the Effect of the oblique Force, is to that of the Perpendicular, as H E to F E; therefore, as the Sine of the Angle of Incidence to Radius. Hence we gather univerſally, that the Force of Percuſſion, or Impulſe of a Body in Motion impinging on any Surface, is as the Sine of the Angle of Incidence, *cæteris paribus.*

The Sum of what has been ſaid, is contained in theſe Propoſitions.

1. THAT, a Force which ſuſtains a Weight upon an inclined Plane, will be to the Weight, as the Sine of the Angle of Inclination of the Plane to Radius. Whence it follows, that if the Power be greater than in that Proportion, it will not only ſuſtain, but move the Weight upwards. Therefore, by the Help of the inclined Plane, one may ſuſtain the greateſt imaginable Weight, with the leaſt imaginable Power, by diminiſhing the Angle of Inclination, ſo that the Weight may be to the Power, as Radius to the Sine of the Angle of Inclination; and by increaſing the Power a little more than in that Proportion, one may not only ſuſtain, but move it.

2. THAT the moſt advantageous Way of applying the Power, is in a Line of Direction parallel to the inclined Plane; and that the Effect of the Power in any other Direction will be to the former, as the Sine-Complement of the Angle of Application to Radius.

3. THAT the Preſſure of the Weight upon the inclined Plane, is as the Sine of the Complement to 90° of the Angle of Inclination.

4. THAT the Force of Percuſſion, or Impulſe of a Body in Motion impinging on any Surface, is as the Sine of the Angle of Incidence.

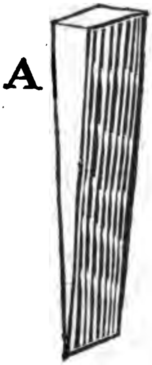
THESE are the moſt remarkable Proportions attending the inclined Plane. It comes in Practice beſides, under two different Forms, the Wedge and the Screw, which come now to be conſider'd.

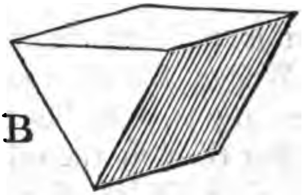
Of the WEDGE.

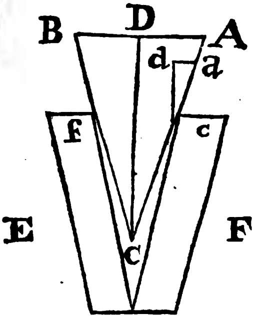
THE Wedge is uſed, as every Body knows, as to ſeparate the Parts of Bodies from each other; all cutting Inſtruments, as Knives, Axes, Chiſſels, *&c.*

&c. being Wedges. It is alſo uſed ſometimes to drive Bodies cloſe together.

THE Wedge is repreſented in the Figure, between two Bodies, in order to ſeparate them. Having by ſome proper Device inſinuated the End of the Wedge C between the Bodies E and F, they drive it in with Blows of







a Mala Mallet. This makes the Wedge ſlide it ſelf between them; which by the Make of the Inſtrument growing wider and wider, makes a Separation as large as is required.

It is plain to common Reaſon, that the Wedge A muſt run more eaſily between any two Bodies than the Wedge B. But then on the other Hand, it will make a much leſs Separation than the other, which enters with more Difficulty. One of them, that is A, produces a leſs Effect, but meets with leſs Reſiſtance; the other produces a greater Effect, but is reſiſted more. Now to know exactly in all Caſes, the Proportion of the Effect to the Reſiſtance, we muſt conſider it in the following Manner.

While the Wedge is driven in gradually farther and farther, between the Bodies E and F, the upper Parts of thoſe Bodies c and f are obliged to ſlide upwards from C to A, and from C to B, along the two Sides of the Wedge. This

Aſcent is made with a Velocity equal to that with which the Wedge inſinuates it ſelf between the Bodies. The Force then which drives the Corner of the Body *c* upwards in any Part of the Wedge, as for Example, from *c* to *a*, may be reſolved into two, *c d* and *a d*, one of which, *a d*, being parallel to A D, has no Effect at all; the other *c d* parallel to C D, having the whole Effect in cauſing the Wedge to aſcend. But *a d* and *c d*, being parallel to A D and C D, it follows, that as *a d* is to *c d*, ſo is A D to C D. But the Force which is effectual in driving in the Wedge, is as *c d*, therefore, alſo, as C D. That Part of the Force which is uſeleſs, is as *a d*, therefore, alſo, as A D.

HENCE we conclude univerſally, that the Reſiſtance made to the Wedge, is as half its Breadth, which is A D; and the Force which drives it in, is as the perpendicular Height, which is in the preſent Caſe C D. And this is the exact Proportion; which tho' it be natural enough to imagine, without a Demonſtration, yet it is more ſatisfactory to ſee it proved.

It is eaſily ſeen, that the Wedge is only two inclined Planes joined together at the Baſe, in which the upper Parts of the Bodies *c* and *f* repreſent the Weights; A C and B C the inclined Planes, upon which the Weight aſcends; the Force which drives in the Wedge, anſwers to the Power ſuſtaining or moving the Weights, and the Reſiſtance which the two Bodies make to the Entrance of the Wedge, anſwers to the Heavineſs of the Weights.

This is the Property of the Wedge; but in Practice, there is ſo ſeldom Occaſion for an exact Computation of the Force of the Wedge, and ſo many Hindrances to its Operation, by Reaſon of the great Friction in this Inſtrument, as makes a Geometrical Enquiry into the Nature of it, a Matter of Curioſity rather than of Uſe.

Of

Of the SCREW.

THE Screw is only an inclined Plane bent round a Cylinder, juſt as a Pair of winding Stairs are the ſame thing with thoſe of more common Uſe. This may be eaſily ſeen by cutting out a right-angled Triangle upon a Piece of Paper, and folding it round a ſlender Stick. One may ſee, that in that Caſe, the Baſe of the Triangle will go round and round the Bottom of the Stick, or to ſpeak in more Mathematical Terms, the Baſe of the Cylinder; the Perpendicular will remain unaltered, and the Hypotheneuſe, which is the inclined Plane, will form it ſelf into a ſpiral Figure like the Threads of the Screw.

THE Nature of the Screw every Body knows to be this. The Thread of the Screw begins at the Bottom and winds about it till it come to the Top; and the finer the Screw is, the greater is the Number of Threads in equal perpendicular Heights. Now the Sum of the Threads, which may be called the Circumference of the Screw, anſwers to the Hypothenuſe in the inclined Plane; and the perpendicular Height of the Screw, to the Perpendicular in the inclined Plane. But in the inclined Plane the Weight is to the Power, as the Hypothenuſe to the Perpendicular. Therefore the Property of the Screw will be,

As the Circumference of the Screw is to its perpendicular Height, ſo is the Weight ſuſtained by it to the Power that ſuſtains it. Hence it follows, that to raiſe a Weight by the Help of the Screw, the Power muſt be increaſed a little above that Proportion.

Therefore, by the Help of the Screw, one may ſuſtain the greateſt imaginable Weight, with the leaſt imaginable Power, by increaſing the Circumference of the Screw, ſo that it may be to the perpendicular Height as the Weight to the Power. And by increaſing the

Power a little more than in that Proportion, one may not only ſuſtain, but raiſe it.

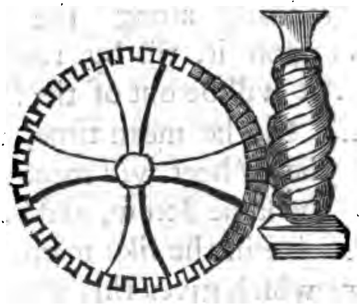
THE Circumference of the Screw may be increaſed two ways, by increaſing the Diameter of the Cylinder, round which the Screw winds; and by increaſing the Number of the Threads. By theſe Means one might produce a Machine able to raiſe vaſt Weights with a ſmall Power, which ſhould take up but little Room.

THIS is true in Speculation; but in Practice the Screw is ſeldom made uſe of to raiſe Weights, for this Reaſon amongſt others, that it is very ſubject to Friction, becauſe all the moving Parts of the Machine are contiguous. One Convenience it has above the reſt; which is, that in raiſing Weights with the other Powers, it is neceſſary, as ſoon as the Weight is once ſuſpended, to keep the Power continually applied to it, otherwiſe if the Power remits its Action before the Weight can reſt upon ſomething elſe to ſuſtain it, the

Weight will deſcend again with great Swiftneſs. But the Screw will give Leave to the Power to remit its Action if Occaſion ſhould be, without that Danger; the Parts of the Machine, by their Conſtruction, ſuſtaining each other.

THE Screw when uſed to impreſs a great Force is generally impelled with a Lever, and the Length of that Lever muſt be conſidered, together with the Force of the Screw alone. The Effect of the Screw alone, will be to its Effect when impell'd by a Lever, as the Semi-diameter of the Cylinder, round which the Screw winds it ſelf, to the Length of the Lever. This muſt be ſo plain to thoſe that have underſtood what goes before, that it is needleſs Labour to prove it particularly.

WHEN the Screw is applied to raiſing Weights, it is generally joined to the Wheel or Axis in Peritrochio, from which Compoſition ariſes a Machine, which is called a *Perpetual Screw.* This Machine is repreſented in the Fig.



but being always to be found in a Jack, an Inſtrument that falls under every ones Obſervation, it will be better known by conſidering it in the Life, than by any Picture. The Nature of it is this. The Screw winds round a Spindle, at the upper Part of which is a Winch to turn the Screw round with. In common Jacks uſed for dreſſing Meat, there is added a Fly at top of the Spindle, to keep the Motion even and regular. The Threads of the Screw are ſo contrived, as in their Revolution to fall in between the Teeth of the Wheel. The Teeth of the Wheel and the Threads of the Screw being made a little inclined to each other, it comes to paſs, that each Tooth of the Wheel impelled by the Screw, aſcends gradually along the Thread which acts upon it, till by revolving a little way, it will be out of the Reach of its Action. In the mean time, another Tooth of the Wheel will meet another Thread of the ſame Screw, and be impelled and revolve in the like manner. And ſo for ever, which gives this Engine the Name of the perpetual or endleſs *Screw.*

In this Engine theſe Things deſerve Obſervation. 1. Each Tooth of the Wheel acted upon by a Thread of the Screw aſcends along that Thread like a Weight upon an inclined Plane. Therefore the Power that ſupports a great Weight by Means of the Wheel only, is to a Power ſuſtaining the ſame Weight, with the additional Help of the Screw, as the entire Weight of any Body is to the Power that ſupports that Body by the Help of an inclined Plane. Now this Power is always leſs than the entire

Weight; therefore, a leſs Power will ſuſtain a Weight by means of the Screw adjoined, than by the Wheel only. The Queſtion then is, to know the exact Proportion of theſe two Powers. And from what has been ſaid, the Anſwer will be the ſame as in the ordinary Screw, *viz.* As the Circumference of the Screw to its perpendicular Height; ſo is the Power that ſuſtains the ſame Weight without the Help of the Screw to the other Power.

Secondly, This Sort of Screw being generally turned round, like the ordinary Screw, by means of a Lever, in ſome Form or other, the Length of that Lever muſt be taken into Conſideration alſo, when we would compute the Effect of this Machine; and the Proportion will be the ſame as that above.

Thirdly, This Screw wants one Convenience, which the other has; the ordinary Screw giving Leave to the Power to remits its Action, without Danger of the Weight's deſcending again; which can't be done with this Sort of Screw.

Fourthly, This Screw by its Conſtruction and Nature, produces in the Wheel a Revolution, whoſe Direction is perpendicular to its own, at the ſame time that it diminiſhes the Power neceſſary to ſuſtain the Weight. For the Revolution of the Spindle is parallel to the Horizon, and the Revolution of the Wheel perpendicular thereto, which Change of the Direction of the Revolution, is of great Uſe upon ſeveral Occaſions.

Sect. III.

General Observations *on the* Mechanical Powers *before deſcribed.*

I Have hitherto diſtinctly and particularly explain'd and demonſtrated the Properties of the Mechanical Powers, and ſhewn the ſeveral Qualities of each of them. I propoſe at preſent to make ſome Obſervations on them, which I thought more proper for this Place than any other. And firſt,

1. I would take Notice of an improper Manner of Expreſſion, often made uſe of, concerning theſe Powers, which are often ſaid to increaſe the Force that is applied to them; whereas (as I hinted before occaſionally) they do not properly increaſe Force, but only ſerve to apply it in the moſt convenient Man-

ner. This is to be underſtood, however, only of the Lever, the Pully, and the Wheel; the inclined Plane depending upon Principles ſomewhat different. To explain this a little. Suppoſe a Stone lying upon the Ground of 300 lib. Weight, and a Man taking hold of it on each Side, endeavouring to lift it, were to apply a Force to it = 300 lib. Then 'tis plain, that the Stone would ſtill remain unmov'd; for the Force of Gravity puſhes the Stone downwards, exactly as much as the Man's Force pulls it upwards. Therefore it can't riſe. But it may then be ſaid, that if the Man ſhould apply to it a Force, never ſo ſmall a Matter greater than 300 lib. as for Example, = 301 lib. that in that Caſe it muſt needs riſe; it being pulled upwards with more Force than 'tis puſh'd downwards by its own Weight. And no doubt if this Force were applied to it continually, it certainly would; but that can't be done for this Reaſon; that when a Man is lifting a Weight in that Manner from the Ground, he applies more Strength at firſt, than he can afterwards. For when a Man endeavours to raiſe a Stone in that Manner, he ſtoops very low, bending the Joints of his Knees, of his Thighs, of his Back-bone, and of his Neck: Now the Sum of the Forces of the ſeveral Muſcles, whoſe Action reſtores thoſe ſeveral Joints to their former State, is the very Force that raiſes the Weight from the Ground, at the firſt Inſtant of its riſing. But by Degrees as the Stone riſes, theſe Muſcles ceaſe their Action one after another; the ſeveral Joints becoming ſtrait, as ſoon as the Weight is raiſed to the ſame Diſtance and Height from the Ground with themſelves, ſo that when the Man has got the Stone as high as his Knees, the Muſcles of the Thighs, which by unbending the Knee aſſiſted him in raiſing it to that Height, now ceaſe to act, and ſo become entirely uſeleſs to him. Now if we ſuppoſe the Action of theſe Muſcles equivalent to 40 lib. it is plain, that here is a Diminution of the Power applied to the Stone; ſo that the Force which at firſt was equal to 301 lib. is now equal but to 261 lib. which being very unequal to 300 lib. which is the Weight of the Stone, can neither raiſe it higher, nor ſupport it at that Height.

BUT if we ſuppoſe this Weight of 300 lib. faſten'd to one Arm of an equal Ballance, or Lever, and a Man preſſing upon the other Arm, with a Force equal to 301 lib. then the Weight may be raiſed by that Force as high as the Length of the Arms of the Ballance or Lever will permit. For the Action of the Muſcles in ſuch a Preſſure, is very nearly ſimilar, and alike to it ſelf, the ſame Muſcles acting all along with much the ſame Force, applied in much the ſame Manner.

SO alſo if we ſuppoſe a Rope faſten'd at one End to the Stone, and the other End thrown over a Pully, and taken hold of by a Man. If this Man pulling the Rope, applies to it at firſt a Force = 301 lib. he will be able to draw the Weight up as high as the Height of the

Pully will give him leave. For in the Action of pulling a Rope downwards, in like manner as in the Action of puſhing one Arm of a Lever, the ſame Muſcles act all along with the ſame Force applied in the ſame Manner. The Wheel being always turn'd either with a Rope like the Pully, or by Preſſure like the common Lever, what was ſaid concerning thoſe two Powers, may be applied to that Power alſo.

2. IT is indeed by this Property, that theſe Machines are generally uſeful in Practice; not ſo much, that they enable us to move a great Weight with a ſmall Power, as becauſe they give room for the Application of a Power to the Weight, which without them could not be applied at all, or at leaſt nothing near ſo conveniently; for in Practice, there is generally not ſo much a want of Hands, as of Convenience for applying them all. To explain my ſelf by an Example. Suppoſe a Cube of Marble, whoſe Side is 2 Foot, or to ſpeak in more vulgar

Terms, ſuppoſe a Piece of Marble 2 Foot every way in Length, Breadth, and Thickneſs; ſuch a Cube would be in Weight = 1296 lib. Let it be required to raiſe ſuch a Stone ſix Foot. Now this could not be done without a Machine, by Strength of Hands only. The Reaſon is, 1. That not above ſix Men could be employ'd about it, that is one at each Side of the Cube; for each Side of the Side being only two Foot, there would not be room for more than one Man. 2. That each of theſe four Men would have 324 lib. each to raiſe; which is a very conſiderable Weight in it ſelf, and farther than moſt Mens Strength goes. And beſides, 3. Suppoſing their natural Strength to be equal, or rather a ſmall Matter ſuperior thereto, yet they would be able to apply but a very ſmall Part of it, to the moving of the Stone, and this again upon two Accounts. 1. Becauſe of the ſlight Hold they would take of it. For they could not all of them claſp it round as a ſingle Man might be ſuppoſed to do, becauſe they would hinder one another; all that each of them could do, would be only to put his Hands underneath the Stone, and ſo all joining together endeavour to lift it at once. But it is a very weak Hold a Man has of any thing, when he only ſupports it with his Fingers open, and has nothing to graſp firmly. Therefore, ſuppoſing each Man's Strength equivalent to the raiſing his Portion of the Weight, yet by Reaſon of the weak Hold he takes, a great Part of it would be of little Service. And then again, 2. Suppoſing their Hold never ſo firm, yet by what was ſaid before, when the Stone is raiſed to the Height of their Knees, a great Part of their Strength would be loſt, by reaſon of the Ceſſation of the Action of certain Muſcles. So that as I ſaid, ſuch a Stone cannot be raiſed by Strength of Hands only; and if it can't be raiſed 6 Foot, much leſs will it be 60 or 70.

BUT, if we ſuppoſe them to apply Levers to this Stone, or if we ſuppoſe the Stone faſten'd with a Rope, and that Rope thrown over a Pully, then it is eaſy to ſee how there may be Force enough applied to raiſe it, by a ſufficient Number of Men preſſing upon the Levers, or pulling the Rope of the Pully, who will alſo, as was ſhewn before, apply their Strength in the moſt convenient Manner.

3. BUT theſe Machines have another Uſe more important; which is, that we may by the Help of them, apply a Power to a Weight, in ſuch a Manner, as to make the Velocities of the Power and the Weight, reciprocally proportional to their ſeveral Quantities; by which Means, as I ſhew'd at large under the former Heads, a ſmall Power may be made to ſupport, and to raiſe a large Weight, which is a very wonderful and important Effect. This was ſhewn largely and particularly before, and therefore I need not repeat it here; but rather make this Obſervation, how the Properties of theſe ſeveral Machines demonſtrate,

monſtrate, *a poſteriori*, that great and uſeful Truth, which I prov'd at firſt univerſally *à priori*, to wit, that in all Bodies mov'd, the *Momenta* are equal, when the Rectangles of the Velocities and Weights into each other, are ſeverally equal.

AND theſe are the Bounds in which Providence hath thought proper to confine the Abilities of mortal Men; ſo that tho' by the Help of Machines, we can indeed make the ſmalleſt Power imaginable, equal, or ſuperior to the greateſt imaginable Weight or Reſiſtance, yet ſtill by how much the Weight exceeds the Power, ſo much the more ſlowly will it move. We may by the Application of Machines lift a Weight of 1000000 lib. with a Power of one Pound only; but then the Weight will move 1000000 times ſlower; in which Caſe, the Power muſt go over a Space of near 200 Miles, before it can raiſe the Weight 12 Inches only. It was a Saying of *Archimedes*, *That if he could have a Place to fix his*

his Machine upon, he alone could move the whole Earth ; and ſo without doubt he could. But one Inconvenience (among others) of ſuch an Attempt, would be this (it is indeed a very great one) that an Engine framed for that Purpoſe, would operate ſo very ſlowly, that not only *Archimedes*, but the Earth it ſelf, would come to an End, before the Effect would be in the leaſt ſenſible.

4. IT is no improbable Opinion, that it was this Property of Convenience of Application, that firſt gave Riſe to the Invention of theſe Machines ; and not any Suſpicion, that one might make a ſmall Power equivalent to a great Weight by Means of them. For if a Man unacquainted with our common Machines, had occaſion to raiſe a heavy Stone lying before him on the Ground, it would be natural for him to apply a Bar of Wood or Iron to it, rather than his own Fingers, becauſe of the Danger there would be of hurting them. And when he had got the Stone upon his Lever,

ver, ſome Chance or other might move him to lay a Prop underneath it, in which Caſe he would find that he would be able to remove the Weight with far leſs Strength than before; and that by puſhing the Prop ſtill nearer to the Weight, he might raiſe a Stone with this Inſtrument with great Eaſe, that otherwiſe he could not move. This, or ſome Chance like it, gave Birth to the Invention of this Power, which being afterwards more nicely taken into Conſideration by the Mathematicians, was found liable to Geometrical Reaſoning and Calculation, and to have thoſe Properties which I have explain'd.

THE Pully ſeems to have taken its Riſe from an Occaſion, ſomething like the following. When Men began to take Delight in building ſumptuous and magnificent Edifices, they found it requiſite, partly by Vanity, and partly by Neceſſity, to make uſe of large Stones and Timbers, at the upper Part of Buildings, at a great Height from the Ground.

Now the Queſtion was, how they ſhould get them up there? To carry them up Ladders upon Mens Shoulders, was always inconvenient and dangerous, and oftentimes impoſſible. The Lever, which was probably one of the firſt invented of theſe Machines, would be, as is ſhew'd before, of no Uſe upon this Occaſion To let down Ropes from the Place deſign'd, and to pull up the Stones by Force of Arms only, was dangerous in all Caſes, and in many impoſſible to be done.

THE firſt Thing probably, that was thought of, was to throw one End of the Rope over a Beam, and to let it down, in order to be faſten'd to the Stone that was to be raiſed; the other End hanging down on the other Side; in which Caſe four or five Men might pull the Rope, which could not be done without ſuch a Convenience. This made it neceſſary, 2. To round off all the Corners of that Part of the Beam over which the Rope went; and to make it as ſmooth as poſſible, becauſe all Corners and Roughneſs there, would cut and fret the Rope. But when all this was done, there would be a great quantity of Friction, or Rubbing of the Rope againſt the Wood, in the overcoming which, a great Part of the Strength employed to raiſe the Weight would be conſum'd and render'd uſeleſs. In order therefore to leſſen this Friction, which ſpent ſo much Force in vain, the Pully was thought of, which giving Way continually to the Action of the Rope, by Reaſon of its turning about its Axis, very much leſſens that Friction. For the Action of turning round upon its Axis has no Effect conſider'd by it ſelf; for if it were poſſible to make the Rope ſlide freely round it, without that Revolution, the Effect would be the ſame as if the Pully turn'd round. But that being phyſically impoſſible, it was neceſſary, that the Pully ſhould turn round upon its Axis, becauſe the Rope will not ſlide ſo freely otherwiſe. After one Pully had been invented, it was natural to make the Experiment of ſeveral together,

ther, by which they found this Machine had a ſecond more extraordinary Property of making a ſmall Power equivalent to a great Weight.

THE Wheel, eſpecially the Compound, appears to have been the Reſult of deeper Thought and Contrivance; but who it was that invented this Power and the Screw, is not certainly known.

THE inclined Plane depends upon a Thought ſo natural and obvious, that it's probable, it was firſt put in uſe, or at leaſt as ſoon as any other. For as we ſee every Day, that Weights puſh'd or drawn up a Hill, will aſcend with the moſt Eaſe where the Hill is leaſt ſteep, it was eaſy to apply this Obſervation to Practice, by making an artificial Aſcent, where a natural one was wanting; in doing which there was this Convenience, that the Aſcent might be made as eaſy as it ſhould be thought proper. If the Stories are true, that we read of the great Works done by the *American* Princes before the *Europeans* got footing amongſt them, it is ſcarce to be queſtion'd, but that they were perform'd by the Help of this Power, all the others requiring in Practice the uſe of Iron, which thoſe People were Strangers to.

THE Wedge is ſo obvious a Thought, that probably 1000 Perſons have invented and made uſe of it without knowing what they did.

4. THO' the Convenience of Application of Force, be probably the Cauſe of the Invention of theſe Powers, yet it is by no Means the chief Advantage of them. For tho' in ſmall Weights, the Conſideration of the Inequality of the Power to the Weight, be of no great Regard; yet in vaſt and enormous ones, it is of very much Conſequence. When only ſmall Weights are to be raiſed, that is, ſuch as the Strength of ſix or ſeven Men is equal to; it is no difficult Matter by the Help of theſe Machines, to

apply the Strength of ſix or ſeven Men, or even of more to the Weight. Now in this Caſe, the Power and the Weight being about equal, there is no Occaſion to examine the exact Quantity of either; for if it ſhould be found upon Trial, that the Hands employ'd are not ſufficient to raiſe the Weight, 'tis an eaſy Matter to employ one, or two, or three Hands more, which we are ſure, before-hand, without any exact Calculation, will be more than ſufficient, only from a tranſient View, and Conſideration of the Bigneſs of the Weight. In a Word, Experience and Dexterity is of more Uſe in theſe Caſes of ſmall Weights, than a Mathematical and ſpeculative Knowledge of the Properties of the ſeveral Powers; and a good Workman will be more ſerviceable than a profound Geometrician.

BUT if, as it ſometimes happens, it be found neceſſary either for Strength, Beauty, or Magnificence to remove Stones or other Bodies of prodigious Weights, then appears the vaſt Advantage,

tage and ſurprizing Effects of theſe Powers, and the Neceſſity of a ſpeculative Knowledge of them. The Advantage of them appears herein, that in vaſt Weights it is impoſſible to apply a Power equal to them, which may be generally done in ſmall ones. Now if it be impoſſible to apply a Power equal to the Weight, either ſome way muſt be found to make a ſmall Power equivalent to a great Weight, or it can't be remov'd at all. Therefore the Uſe of theſe Machines is very great, becauſe by the Help of them, one may make a ſmall Power equivalent to a great Weight, and by that Means lift or remove Bodies, which without their Aſſiſtance cou'd not be ſtirred.

BUT in order to find a Machine, which ſhall remove a vaſt Weight with a Power much leſs, a previous Calculation is abſolutely neceſſary. The firſt Thing to be done, is to find out either exactly, or pretty nearly the Quantity of the Weight, which is not ſo neceſſary,

where

where the Power and Weight are nearer to Equality. If it be a Stone or ſolid Body, that is to be moved, the beſt Way of doing this, is to find the Weight of a cubical Inch or Foot of ſuch Body. This being had, the next Thing, is to find the ſolid Content of the whole in cubical Inches or Feet, according to that Sample. For Example. Let it be required to raiſe a Stone 18 Foot in Height, 7 in Breadth, and 4 in Thickneſs, and that by reaſon of ſome Hindrances, too various to be particularly accounted for, one could not apply to it more than the Force of 100 Men. The Queſtion then is, how by the Principles before laid down, and by the Machines above deſcribed and explained, one ſhall be able to perform what is required, and what the Steps are, which muſt be taken to that End.

THE firſt Thing then to be done, as I ſaid, would be to find the Weight of a cubical Inch or Foot of this Stone. Now if we ſuppoſe it to be of Marble, as we may for Calculation ſake, it will be found, that a cubical Inch will weigh about 6 oz. $\frac{1}{2}$. Let it be ſuppoſed ſo. From thence it will follow, that a cubical Foot containing 1728 cubical Inches, will be in Weight = 6 oz. $\frac{1}{2}$ multiply'd by 1728, that is to 11232 Ounces, that is to 702 Pounds, the Weight of a cubical Foot. This being found out, the next Thing is to find how many of theſe cubical Feet, that is how many times 702 Pounds are contained in the whole Stone. Now to find how many cubical Feet are contained in the whole, we muſt multiply, as every one knows, the Height, Breadth, and Thickneſs into each other; that is, 18 Foot into 7, which makes 126, and that again into 4 which makes 504, the Number of cubical Feet in ſuch a Stone. If therefore one of theſe cubical Feet weighs 702 Pounds, the Weight of the whole will be = 702 x 504, that is = 353808 Pounds. Let us take a round Number, and ſuppoſe the Weight equal to 360000 Pounds. Now to remove this Weight, it was ſuppoſed, that we had only the Strength of 100 Men to employ. Let the Strength of thoſe Men be ſuppoſed ſeverally equal to 200 Pounds each. Multiply that Number into 100, and the Product will be 20000, the Sum of all theſe Mens Strength. So that here is given a Weight = 360000 Pounds, to be raiſed by Means of a Power equal only to 20000. Let us now conſider of the ſeveral Ways by which this Weight may be raiſed by this Power, and endeavour to find, which of them will be the moſt eaſy and practicable.

THE firſt thing neceſſary, is to find the Proportion of the Weight to the Power; which is done by dividing the greater by the leſs, that is 360000 Pounds, the Quantity of the Weight, by 20000, the Quantity of the Power, the Quotient whereof is 18; which ſhews, that the Power is to the Weight, as one to 18.

THIS being known, we ſhall examine the ſeveral Powers one after another.

THE Lever is the firſt that offers it ſelf to our Conſideration. It was ſhewn in the Diſcourſe upon that Power, that if the Length of that Arm of the Lever, which is next to the Power, be to the Length of that next the Weight, as the Quantity of the Weight to the Quantity of the Power, that is in reciprocal Proportion; that then the Power would ſupport the Weight; or if it were yet longer, it would not only ſupport, but raiſe it. Therefore to apply that Theorem to the preſent Caſe, it may be ſaid, that if a Lever be made, and its Length be divided in ſuch ſort, as that the Diſtance from the Prop to the Weight be equal to one, and from the Prop to the Power be equal to 19, that then this Power of 20000 Pounds applied to it, would raiſe the Stone. But the Impoſſibility of doing any thing with that Machine appears at firſt Sight. For ſuch a Lever as will make a Power = 1, equivalent to Weight = 18, muſt be at leaſt 57 Foot long; and where can ſuch a Piece of Timber or Iron be found of ſufficient Strength to endure ſuch a prodigious Bearing, as it muſt have at the greateſt Diſtance from the Fulciment? How can it be ballanced upon the Fulciment? or ſuppoſing it might be done, it muſt coſt as much Trouble to do that, as to remove the Weight. Beſides one Lever alone would not do, and how could more be applied to it, to co-operate exactly with the other? But it is needleſs to ſpend Words in a plain Caſe. Therefore the Lever muſt be laid aſide as uſeleſs for the preſent Occaſion.

THE Pully comes next in order under Conſideration. And this promiſes better than the Lever. It was ſhewn before, that a Weight ſupported by the Help of Pullies, will be to the Power, as the Number of Strings applied to the lower Pullies, is to Unity. Now from this Principle, it is eaſy enough to determine, how to raiſe this Stone by the Help of this Machine. For by that Principle,

ciple, if we ſuppoſe Tackles applied to the Weight, conſiſting of 9 lower and 9 upper Pullies, it will follow, that the Weight may be ſupported with thoſe Pullies by a Power that is to it, as one to 18. For the Power that ſuſtains the Weight, is to the Weight, as one to the Number of Strings applied to the lower Pullies, that is in this Caſe as one to 18, there being 18 Strings applied to 9 Pullies; that is, one on each Side. But the whole Weight was ſhewn to be the whole Force applied to it, as one to 18. Therefore if we ſuppoſe, ten ſuch Tackles applied to ſuch a Stone, and that ten Men be apply'd to each of them, it follows, that the Stone will be acted upon by a Power ſufficient to ſuſtain it. For then each Man's Force ſo applied, will be equivalent to 18 times the Quantity in the Weight; that is every Man's Force being by our Suppoſition = 200 lib. that 200 lib. will ſupport 18 times as much in the Weight; that is, 3600 lib. which will be the Quantity of the Weight ſupported by one Man. Now if we multiply 3600 lib. the Quantity of Weight ſupported by one Man, by 100, the Number of all the Men, the Product will be 360000, the Weight of the entire Stone.

But it muſt be conſider'd, that a Power that moves, or raiſes a Weight, muſt be greater than that which ſuſtains it; for which Reaſon we muſt increaſe ſtill more the Effect of the Power in order to do that which is required. Now if we ſuppoſe only one lower and upper Pully, added to each of the Tackles, ſo as to make Ten upper and lower Pullies, inſtead of Nine, the Number before; we ſhall increaſe the Effect of the Power a tenth Part, which will be abundantly ſufficient for the Purpoſe deſigned.

The Wheel, as I took Notice of in my Diſcourſe upon it, is of two Kinds, Simple and Compound. The Simple comes under two remarkable Forms, the Crane and the Capſton. In the Cranes uſed for lifting Goods out of Veſſels, every

very one muſt have obſerv'd the Wheel, within which a Man continually walking cauſes it to turn round. Through the Centre of that Wheel there paſſes a Beam, round which a Rope winding it ſelf, at one End, cauſes the Weight faſten'd to it at the other End to aſcend; till ſuch a time as it is raiſed to the deſigned Height; and then by pulling the Crane round, it is brought aſhore. Now this Wheel is the very ſame thing with that which I then deſcribed, and called the *Simple Wheel*; and I there ſhewed, that a Weight ſupported by that Machine, would be to the Power ſupporting it, as the Radius of the Wheel to the Radius of the Axle round which the Rope winds.

BUT this Engine would be of little Service in the preſent Caſe, for this Reaſon amongſt others, that only one or two Men can work within each of them, and how to make, or manage, or direct ſuch a Number, as would be neceſſary of the Spindle, makes 6 Foot, which is what in theſe Caſes is the Radius of the Wheel, as was ſaid before. Then it will be as 6 to 72, that is, as 1 to 12; ſo is the Quantity of a Man's Strength to the Weight he is able to ſupport by one Bar of this Machine. Therefore, as 1 to 12, ſo is 200 lib. to the Weight each Man ſuſtains, that is, to 2400 lib. Now this multiply'd into 100, produces only 240000 lib. which is much leſs than the Weight of the Stone. Therefore with this Machine alone, ſo applied, the Stone cannot be raiſed. The beſt and moſt eaſy Remedy for this Defect, is to make the Rope of each Capſton be faſten'd to a double Pully; by which Means the Effect of each Man's Strength will be double to what it was before, that is = 4800 lib. which multiply'd into 100, gives 480000 lib. the Weight which theſe Mens Strength, by the Help of this Machine, is able to ſuſtain. Now if their Strength be able to ſuſtain 480000 lib. it will be much more than ſufficient to raiſe only 360000.

IT was chiefly by the Help of theſe Capſtons, that *Domenico Fontana* an *Italian* Architect removed the great Obelisk in the Centre of the Portico before St. *Peter*'s Church at *Rome*; whoſe Weight is very near 1000000 Pounds.

THE Compound Wheel may be made uſe of alſo in this Caſe, with great Convenience. If we ſuppoſe a Wheel, whoſe Diameter is as *6* to have a Pinion at the other End of the Axle, whoſe Diameter is as 1; and that Pinion to impel a Wheel, whoſe Diameter is to that of the Axle, as 3 to one; then by what was ſhewn before, if a Rope were to be thrown over the firſt Wheel, and pull'd by a Man, his Force would be equivalent to 18 times the Quantity in the Weight; and therefore if we ſhould ſuppoſe ten ſuch Machines applied to the Weight in Queſtion, and ten Men working at each of them, it will follow, that the Weight will be acted upon with a Force ſufficient to ſuſtain it. But ſince a

Force that ſuſtains, is leſs than that which raiſes, it follows, that by increaſing either of theſe two Proportions, there will be produced a Machine ſufficient not only to ſuſtain, but to raiſe the Stone.

THE inclined Plane will not be ſo convenient as the three Machines before mentioned, upon theſe Accounts. 1. Becauſe, theſe three Methods are equally practicable, whether the Weight be to be raiſed to a great or ſmall Height, whereas the inclined Plane cannot be made uſe of at a great Height. 2. Becauſe, in order to fit ſuch a Stone for being drawn up the inclined Plane, it muſt be incloſed in ſome proper Contrivance to enable it to roll upwards, which its own natural Figure will ſeldom allow. Now this Contrivance, of whatever Kind, will be a great additional Weight, and therefore the Uſe of it is to be avoided.

THE Wedge is not uſed for the raiſing of Weights; and tho' the Screw might be ſerviceable upon ſome Occaſions, yet at preſent it will be of no Help at all by it ſelf; tho' under the Form of a perpetual Screw, it may be applied to the Wheel or Axis in *Peritrochio* with good Advantage, as was obſerved in the Diſcourſe of the Screw.

5. ALL that has been ſaid hitherto of theſe Powers depends upon the Suppoſition, that the Machines made Uſe of, are perfect in their Kind; that is, that the Levers be homogeneous and of equal Breadth and Thickneſs in every Part: The Pullies perfectly round, and turning with all imaginable Freedom round their Axes; and the Wheels and Axles in like manner upon theirs; that the inclined Planes be every where perfectly ſmooth, and even; the Wedges made of ſuch Matter as to be perfectly ſmooth and lubricous; and the Screws in like manner perfectly ſmooth, and ſo fitted to each other, as to turn with all imaginable Freedom. So far as the Machines fall ſenſibly ſhort of this Perfection; ſo far will

the Effect of the Machines differ from the Calculation.

THE moſt remarkable Imperfection is, that which is called the Friction or Rubbing of the Parts of the Machine, in overcoming of which, a great Part of the Strength is loſt. This Friction is occaſioned by Roughneſs of the contiguous Surfaces, which is generally produced in Length of Time, and ſometimes proceeds from other Cauſes. So old Hinges of Doors, or Window-Shutters, that have not been ſtirred, or opened a long time, move ſtiff. So if we have Occaſion to open a ruſty Lock with a ruſty Key, we find it hard to overcome the Reſiſtance the Key meets with, and to turn it thro' the Wards, tho' the Lock, abating for its Ruſtineſs, be in as good Condition as ever. So Screws of any Kind, that have lain by a long time, are generally turned with great Pains; becauſe the Air operating upon them all that while, by ruſting Metal, or rotting or ſoftening Wood, makes the contiguous

Surfaces rough; which Roughneſs we find by Experience is very hard to overcome. Now the Reſiſtance made in all theſe Caſes, ariſing from the irregular Situation of the Parts, is what we call Friction. The moſt ſmooth and poliſhed Bodies in the World have a little, tho' in ſome, it is ſcarcely ſenſible; and if the ſmootheſt have ſome Friction, thoſe Bodies that are leſs ſmooth muſt have more. The Parts of Machines conſiſting chiefly of Iron, Wood and Ropes, muſt therefore have ſome Friction; which is increaſed in time by the Iron growing ruſty, the Wood ſoft and rotten, and the Ropes hard and ſtiff.

IT will be proper to conſider the ſeveral Powers, and to examine how far each of them are ſubject to this Inconvenience.

THE Lever is ſubject to little or no Friction, as ballancing round a Point in the Theory, and a very ſmall Surface in the Practice.

THE Pully is ſubject to Friction from the Ropes that go round the Blocks, and the Axles turning about the Centers. For this laſt Reaſon, it is convenient to have the Pullies as large as one can get them; becauſe in a large Pully, the Rope acts at a greater Diſtance from the Centre where the Friction is; and by that Means has the greater Force to overcome it.

THE Wheel is ſubject to Friction, by its preſſing with all its own Weight, and that of the Weight it ſuſtains, upon its Supports, tho' by proper Contrivances this Inconvenience may be much alleviated. One Obſervation may be properly inſerted here; that as the Weight which is ſuſtained or moved by a Multiplication of Wheels, makes a very different Impreſſion upon each of them, ſo it is of great Conſequence in Practice to make the Axle, which immediately bears the Weight, and the Wheel belonging to that Axle ſtronger and more ſubſtantial than any of the other Axles or Wheels, which may be made ſo much ſlenderer, as they are farther removed from that Axle and Wheel, at which the Weight hangs.

THE inclined Plane is not ſubject to any Friction at all, but is attended with this Inconvenience; that if the Weight to be moved is not naturally of a ſphærical or cylindrical Figure, it is neceſſary to annex to it ſome Contrivance to make it capable of rolling upward; which additional Weight may be perhaps equivalent in ſome Caſes to Friction.

THE Wedge and Screw are both ſubject to a great deal of Friction, becauſe the moving Surfaces are contiguous. For this Reaſon a particular Care muſt be taken to keep theſe Machines in good Order, and to make them of ſuch Materials, as may be leaſt liable to this Inconvenience.

Of the SAIL.

THE Sail being of ſo great and univerſal Uſe, by its Application to the Motion of a Ship, and of a Windmill, it is neceſſary to add an Explanation of its Nature and Properties, to what has been ſaid concerning the other Mechanical Powers. The moving Principle of this Power being that Motion of the Air which we call Wind, and not the Strength of an animal Creature, as of the Powers above deſcribed, it will be proper to ſpend a few Words in the Conſideration of this new Principle.

WIND is a Stream or Current of the Air, tending ſometimes one Way, ſometimes another, in all manner of Directions. The Force of it is ſometimes prodigious, as every one knows; and from that Height of Violence it deſcends through all Degrees of Strength and Weakneſs to a perfect Calm. Its Velocity in furious Tempeſts is found to be about 60 Miles in an Hour; and ſometimes its Motion is ſo ſlow, that one may out-walk it. Now between theſe two Extreams there are ſeveral intermediate Degrees of Velocity, which ſerve very conveniently for a Principle of Motion, when their Force is applied and directed to a certain End by proper Contrivances. This is done by Means of the Sail; which is in Practice a large Sheet of Canvas, or ſuch like Matter; and in Speculation, a Mathematical Superficies.

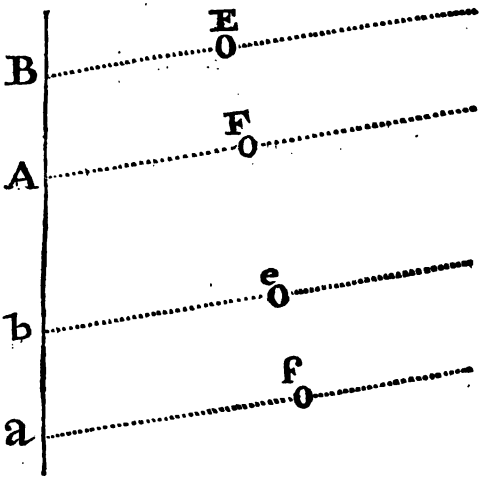
THE manner of the Operation of the Wind upon the Sail, may be I think beſt conceived by a Suppoſition like the following. Let us imagine a Ship in the Middle of the Sea, and the Air to be perfectly calm. Let there be ſuppoſed afterwards a Stream of Air to come on the ſudden from the North, for Example, or any other Point of the Compaſs. Let the Motion of this Stream of Air be of an ordinary Swiftneſs, as for Example, 10 Miles an Hour. This Current of

Air will move for a great way along the Surface of the Sea, without any Interruption; its ſeveral Particles deſcribing Lines parallel thereto. At laſt, when it is arrived at the Place where the Ship is, a Portion of this Stream of Air, whoſe Area and Figure is equal and ſimilar to that of the Sail, will be intercepted by the Sail in its Courſe. This intercepted Portion will endeavour to drive the Sail, and conſequently the Ship before it with a Velocity equal to its own. And indeed were it poſſible for the Ship to float upon the Sea without drawing Water, it actually would do ſo. But ſince by the Laws of Hydroſtaticks, and the Structure of the Veſſel, the Keel muſt be immerſed under the Water, it comes to paſs, that the Keel, in dividing the Water for its Paſſage, meets with a good deal of Reſiſtance; and that Reſiſtance, though it is not ſufficient, as we find by Experience to ſtop the Motion of the Ship, muſt to be ſure in ſome Meaſure retard it. So that by the Action of the Wind upon the Sails, the Ship is carried along; but by Reaſon of the Reſiſtance the Water makes to the Keel, not ſo faſt as the Wind it ſelf moves. After this Manner, we may form an Idea of the Operation of the Wind upon a Ship at Sea.

As in the other Powers, ſo in this, many Truths relating to it may be diſcovered groſly and in general from ordinary Obſervation; but our preſent Deſign being to conſider it more nicely and Mathematically, we ſhall now proceed to lay down ſome Axioms, as Premiſes, from whence to draw ſuch Concluſions, as may ſhew the Properties of this Mechanical Power.

I. In the Current of the Air, which we call Wind, the Particles of the Air will naturally be born along in right Lines parallel to each other. It is true, we ſometimes ſee it otherwiſe; as when the Atoms of Duſt are dancing about in the Sun, they move in irregular Figures, as the Particles of Air are carried about: But that proceeds always from ſome Obwhich makes the Stream of Air recur into it ſelf: Whereas, if that Obſtacle were removed, thoſe little Motes would be ſeen to proceed in right Lines parallel to each other, or nearly; the Irregularity of the Air's Motion it ſelf keeping them from being exactly parallel. However, in our Reaſonings about them, we always ſuppoſe them parallel, the Difference being of little or no Conſequence.

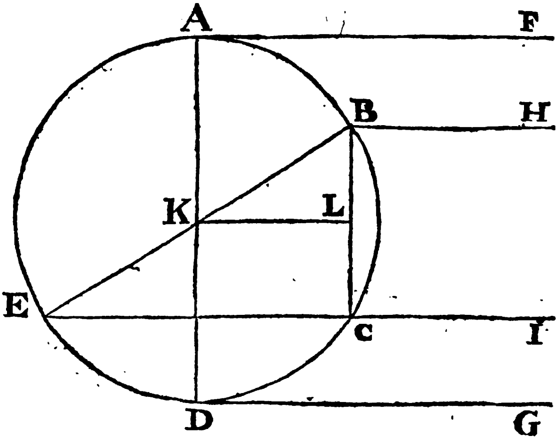
II. These Particles of Air being alike in the Direction of their Motion, muſt be ſuppoſed alike alſo in other Reſpects; ſo that what is found true of any one Part or Particle, may be alſo applied



to the reſt. As if B A *b a* repreſent a Superficies, which is a Sail mathematically conſidered, and a Particle of Air at E moving in the Direction E B, impel the Point B of the Superficies with a given Force; then by this ſecond Axiom the Particles F, *e*, and *f* moving in Lines parallel to E B, will impel the Superficies in the Points A, *b*, and *a* in the ſame manner, and with the ſame Effect in all Reſpects as the Particle E did in B. So again, if in the Space between E B and F A, F A, we ſuppoſe a great Number of Particles moving in the Direction E B towards the Part B A of the Superficies and impelling it, each of thoſe Particles, will impell the Superficies ſucceſſively in the ſame Manner. And again, the Effect of the Aggregate or Sum of all thoſe Impulſes will be the ſame as of another Set of Particles moving in the ſame Direction between *e b* and *f a*, and impelling an equal Part of the ſame Superficies *b a*.

Since therefore all the Particles of Air moving in the ſame Direction have like Effects, it follows, That a Stream of Air or Wind operates upon a Superficies or Sail, in Proportion to the Sine of the Angle of its Incidence. For by the ſecond Axiom the Effect of the whole Stream is like to that of any one Particle. And by what I ſhew'd under the inclined Plane, the Effect of any one Particle will be as the Sine of the Angle of Incidence. Therefore the Effect of all the Particles, that is, of the whole Stream of Air, will be alſo as the Sine of the Angle of Incidence.

BUT this is true upon this Suppoſition only; that the Number of airy Particles acting upon the Sail or Superficies, is the ſame at all Degrees of Inclination. If this Suppoſition be not true, there will ariſe an Alteration of that Proportion. For if in one Poſition 1000 Particles impel the Sail in a certain time, and in another Poſition only 750; 'tis plain that all other things being alike, the Action in one Caſe muſt be as 1000, and in the other Caſe as 750 only. To find therefore the Difference of the Impulſe occaſioned by the different Situation of the Sail, let there be deſcribed in the Fig. the Circle A B C D E. Draw the



two Diameters A D, B E. Let the Stream of Air proceed parallel to the Tangents A F, D G, and let a Surface or Sail, intercepting that Stream of Air, be repreſented by the Diameter A D, ſtanding at right Angles to the Tangents A F, D G, and therefore to the Current of Air. Then 'tis plain, that the whole Portion of this Stream contained in the Space between thoſe Tangents, will fall upon the Sail repreſented by the Diameter A D.

LET us now ſee what will be the Conſequence of turning the Sail in an oblique Situation, as B E. Draw B H, E I parallel to A F, D G. Then it plainly appears, that the Particles of Air contained in the Space between B H and A F on one Side, and in the Space between C I and D G on the other, paſs beſide the Sail B E when it is in this Poſition, not touching it at all. Draw B C parallel to A D, and then the Portion of the Stream acting upon the Sail B E will be as B C, and the Portion of the ſame Stream acting upon the Sail A D, will be as A D it ſelf. Draw K L from the Centre K parallel to I E. Then is the Angle B K L, equal to the Angle of Incidence I E B. Therefore B L is the Sine of the Angle of Incidence. But the two different Portions of the Airy Streams acting upon A D and B E were ſhewn to be as A D and B C; therefore they are alſo as the Halves of thoſe Lines; that is, as A K the Radius, and B L the Sine of the Angle of Incidence. That is univerſally, the Number of Particles falling upon the Sail, will be as the Sine of the Angle of Incidence.

Now this Ratio compounded with the former, makes the entire Effect of a Stream of Air upon a Sail, to be as the Square of the Sine of the Angle of Incidence.

But this is ſtill upon a Suppoſition, that the Velocity with which the Particles of the Air move, is always the ſame. But that is not always the Caſe. Now 'tis plain, that the Impulſe of a Stream of Air, moving with a Velocity, that is as 2 will be twice as great as that of a Stream moving with a Velocity, that is as 1. Therefore the Impulſe of the Wind upon a Sail, will be as the Square of the Sine of the Angle of Incidence, and as its Velocity alſo.

But this Proportion ſuppoſes the Number of Particles of Air impelling the Sail to be the ſame at all Degrees of Velocity, which is not true.

For the greater the Velocity of the Motion of the Air is, the quicker is the Succeſſion of the airy Particles impelling the Sail. Therefore the Number of thoſe Particles is as the Velocity.

THEREFORE, ſince the Effect of the Wind is as the Velocity of each Particle, and as the Number of Particles, which is alſo as the Velocity of each Particle, it follows, by compounding theſe two Proportions together, that the Effect of the Wind is as the Square of its Velocity, and as the Square of the Sine of the Angle of Incidence.

THE next Thing to be conſider'd, is the Direction of the Operation of the Wind, or the Line or Lines of Directions, in which a Stream of Air impinging upon a Superficies, endeavours to impel it. Now if we ſuppoſe the Wind to move in a Direction perpendicular to the Superficies, it is evident, that it will impel it along the ſame right Line in which it ſelf moves. As if a Wind moving in the Direction A B impel a Superficies, as C D at right Angles, as in Fig. 'tis plain that it muſt oblige it to move along, and parallel to the Line B E. But if we ſuppoſe the Line A B, or the Direction of the Wind to be inclined to the Superficies, 'tis not ſo clear, which Way the Superficies will be impell'd. To determine that, let there be drawn the Line F B perpendicular to the Surface, and the Line A F parallel thereto. Then will the Action of the Wind upon the Surface be reſolved into two, one parallel to the Surface, and therefore of no Effect, and the other perpendicular thereto, parallel to F B. Now this laſt Force tends to impel it along B E, with a Force, that is, to the Force of the ſame Wind at Right Angles, as F B to A B. Therefore the Superficies will move in the ſame Line of Direction when impell'd at any Angle, and the Difference will conſiſt only in the Velocity of its Motion.

FROM

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Fig. I.

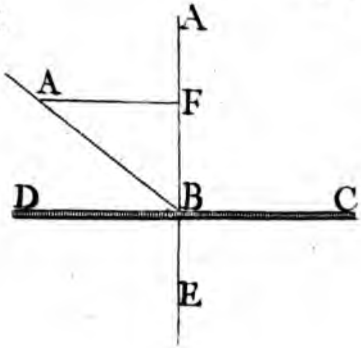
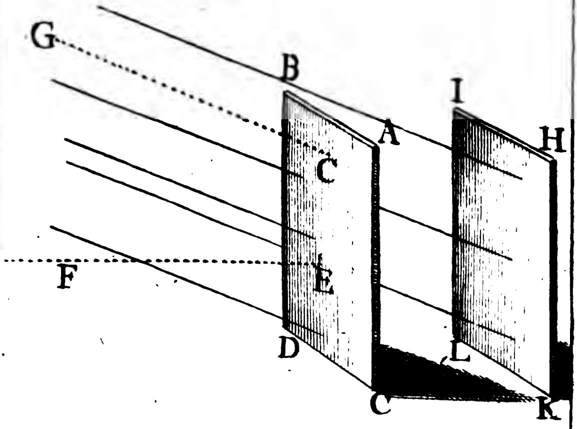


Fig. II.



FROM what has been ſaid, we may deduce theſe Concluſions relating to this Power. 1. That ſince the Effect of the Wind is *cæteris paribus*, as the Square of the Sine of the Angle of Incidence, it will be neceſſary, in order to make its Effect as great as poſſible, to make that Angle as great as poſſible, that is a right one. 2. That nevertheleſs, this is to be underſtood only of a ſingle Sail; but if as in large Veſſels, we make uſe of more than one, the Caſe will be much alter'd. For in the Figure let A B C D, H I K L, repreſent two parallel Sails, which we may ſuppoſe applied like thoſe in a Ship; and imagine the Wind to move in the Direction E F, at right Angles to the Superficies A B C D. Now in that Caſe, by what was ſaid before, it will undoubtedly have a greater Effect upon that Sail, than if it mov'd in the Direction C G at an Angle of 60°, by how much the Square of Radius exceeds the Square of the Sine of 60°. But then the hindmoſt Sail will be of little or no Uſe, by reaſon that the other ſtanding directly between it and the Wind, will keep off the Wind from acting upon it. But if the Wind moves at an Angle of 60° to both the Superficies, it will act equally upon them both, with an Effect, that is as the Square of the Sine of 60°. Therefore their ſeveral Forces being united, will exceed the Force of a Wind applied at right Angles, as much as twice the Square of the Sine of 60° exceeds the Square of Radius. And if we ſuppoſe three Sails behind each other, as is uſual, the Advantage of ſuch a Side-wind appears ſtill greater; being as three times the Square of the Sine of 60° to the Square of Radius. 3. The Effect of the Sail being in Proportion *cæteris paribus* to the N° of Particles, and the N° of Particles in Proportion to the Bigneſs of the Sail, or the Area of the Superficies, it follows, that the Effect of the Sail, *cæteris paribus*, will be as its Area; ſo that in order to make its Effect as great as poſſible, its Area muſt be as great as poſſible.

I have hitherto ſpeculatively conſidered the Properties of the Sail. I could have been very glad to have taken Notice diſtinctly and particularly of its Uſe and Application in the Management of a Ship. But not having had an Opportunity of ſeeing the Practice, as having never gone a Voyage by Sea, it would be little better than Impertinence, to pretend to ſay any thing upon that Subject. I ſhall only at preſent make an Obſervation or two upon its Uſe upon other Occaſions; the firſt of theſe is a Windmill, in the Contrivance of which, there is ſomething more curious than perhaps every Perſon has obſerv'd.

THE Difference of the Sails of a Ship and of a Windmill, conſiſts in this; that the former proceed continually in a right Line before the Wind, whereas the latter have a circular Motion round a fixed Point or Axis; which circular Motion is always at right Angles to the Current of the Wind, or nearly ſo; otherwiſe the Sails can't turn.

To ſhew how this Effect is produced, for it ſeems at firſt Sight a little paradoxical, to cauſe Sails to move in a Direction perpendicular to that of the Wind, we will at firſt ſuppoſe in the Figure, one Sail, only as S, faſten'd to an Axis A B, which Axis is to be made to turn by means of the Wind upon the Sail S. To this Purpoſe the Sail muſt be placed, not perpendicular to the Courſe of the Wind, becauſe the Wind, tho' it has then the greateſt Influence upon it, has no Effect in making it turn round; nor parallel to the Current of the Wind, becauſe in that Caſe the Wind could not act upon it at all; but obliquely, ſo that the Wind moving in the Direction A B, A B, A B, A B, may fall upon the Sail aſlant. In this Caſe, the Effect will be as the Sine of the Angle of Incidence, which Effect, at an Angle of 45°, is very conſiderable. The Sail thus impell'd by the Wind, is oblig'd to turn round where

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Fig. I.

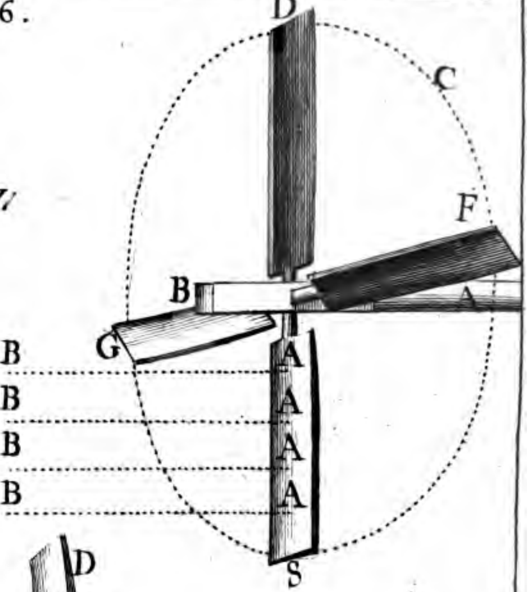
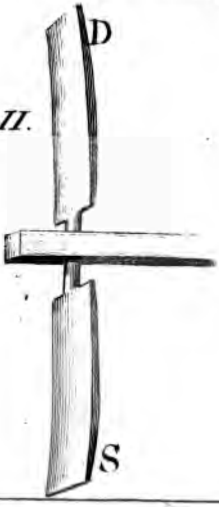


Fig. II.



where it finds Liberty ; but by the Conſtruction of the Machine, it can only move towards F, round the Axis A B, which it accordingly does continually ſo long as the Wind holds.

Now if the oppoſite Sail D were in the ſame Poſition as the Sail S, as in Fig. the Wind would act upon the Sail D in the ſame manner as it does upon S, and cauſe it to revolve towards C in like manner, and with the ſame Force. But theſe two Motions equal and contrary, would deſtroy each other, and ſo render the Machine uſeleſs. Therefore to prevent that, they turn the Sail D in a contrary Poſition, tho' at a like Degree of Inclination ; by which Means, at the ſame time that the Sail is impelled by the Wind, and forced to revolve towards F ; the Sail D is impelled by the Wind falling upon it in the ſame Angle, and forced to revolve towards G. Now this Revolution of the Sail D conſpires with that of the Sail S ; which cauſes the Axis to turn round with twice the

Force.

Force. In the ſame manner, one might ſhew, that the Sail F will revolve towards D, and the Sail G revolve towards S, ſo that all the Sails will conſpire to move in the ſame Direction.

Secondly, Upon this Power depends the Mechaniſm of the Flying of Birds; the Wings of the Bird anſwering to the Sail, and the Reſiſtance the Air makes to them in their Motion, anſwering to the Impreſſion of the Wind upon the Sail. I ſhall make an Obſervation or two upon the Structure of thoſe Inſtruments of Motion, tending to illuſtrate the Wiſdom of the great Creator in the Contrivance and Fabrick of them.

AND *Firſt*, ſince the Action of the Wind upon a Superficies, is greater or leſs, as that Superficies is greater or leſs; ſo in this Caſe we find the Wings of a Bird, eſpecially when ſpread out, are very large in Proportion to the Body to be ſuſtained by them. *Secondly*, the Wings are alſo made a little Concave on the lower Side; becauſe the Reſiſtance the Air makes to them, muſt needs be ſo much the greater, as being ſurrounded and confined under that Cavity of the Wing; and by that Means, not eſcaping ſo ſoon, it undergoes, as it were, a ſhort Condenſation, which increaſing the Number of airy Particles acted upon by the Wing, muſt increaſe the Reſiſtance made to it. *Thirdly*, as the Reſiſtance made to the Wings in their Deſcent, ſhould be as great as poſſible; ſo the Reſiſtance made to them in their Return upwards, ſhould be as ſmall as poſſible; becauſe if theſe two Reſiſtances were equal, the Bird would deſcend as much when the Wings are lifted up, as it riſes when they are let down. To this End, as in the Deſcent, two Contrivances are made uſe of to increaſe the Superficies of the Wing, and the Reſiſtance made to it, ſo in the Aſcent the ſame Contrivances are reverſed. For as the Wings are Concave on the lower Side, on the upper Side they are convex, by which Means, the Air ſlides off with greater Eaſe. And as the Bird has a Power of expanding the Wings in their deſcent beyond their natural Breadth, in order to receive as great a Quantity of Air under them as poſſible; ſo in their Aſcent it has a Power of contracting them as much, to receive as ſmall a Quantity of Air as poſſible. Some Birds have a Faculty of turning the great Feathers in their Wings to a perpendicular Situation, when they lift up their Wings. By this Means the Air having free Paſſage thro' the Intervals of the great Feathers, the Reſiſtance it makes to this Motion of the Wings muſt be very ſmall. A like Contrivance is obſervable in Inſects, in whoſe Wings are ſeen, as it were ſmall Threads branching themſelves quite thro the Wing. By the Help of theſe Threads opening and ſhutting, the Wing is made to open and contract it ſelf, like a Ladies Fan by Means of the Sticks. And theſe two oppoſite Actions are performed with incredible Swiftneſs; the Motion of the Wings, in Inſects eſpecially, upwards and downwards, being ſo quick as to elude

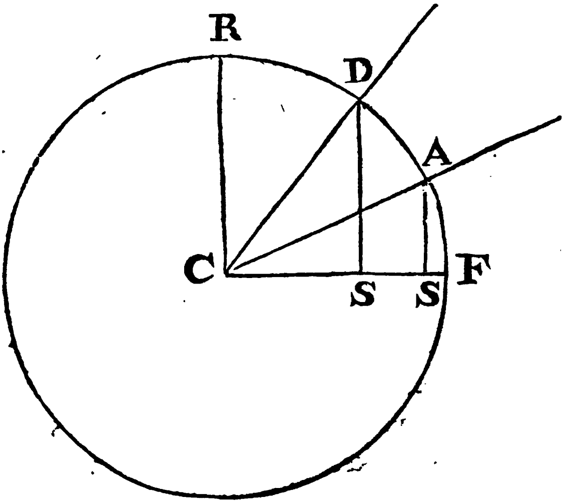
lude our Sight. And yet in every one of theſe quick Motions, each of theſe oppoſite Actions of dilating and contracting the Wing is certainly performed.

APPENDIX.

I Believe the Reader, tho' unskill'd in Geometry, will find very little in the former Part of this Work relating to the Laws of Motion, and the four firſt Powers, which a tolerable Skill in Arithmetic, and a little natural Sagacity joined with ſome Application will not make him Maſter of. At leaſt if he finds ſome Difficulty to enter into a few of the Demonſtrations, in which it was impoſſible to avoid ſome Geometrical Terms and Reaſoning; he will however eaſily underſtand the Concluſions drawn from them, ſo as to take them upon Truſt till he has attained Geometry enough (and it is a very little, that is requiſite) to go through the Proofs afterwards. But becauſe in treating of the other Powers, it was neceſſary to expreſs not only the Reaſonings, but ſometimes alſo the Concluſions ariſing from them in Terms purely Geometrical, which would make all there ſaid of no Service to thoſe, who are unacquainted with the Meaning of thoſe Terms, as I ſuppoſe ſeveral of my Readers to be; I ſhall therefore ſubjoin, by way of *Appendix* ſome numerical Calculations founded on thoſe Concluſions. Becauſe ſuch Calculations will ſerve not only to make the Principles on which they depend, better underſtood; but to direct others in making the like, when Occaſion ſhall require.

THE firſt Propoſition relating to the inclin'd Plane (*V. pag.* 122. *or* 129.) is that a Force, which ſuſtains a Weight upon an inclined Plane, will be to the Weight it ſelf, as the Sine of the Angle of Inclination of the Plane to the Radius. But this Term of the Sine of an

Angle being new, it is neceſſary to explain, *Firſt*, what it means in general, and *Secondly*, the Meaning of it as it ſtands in this Propoſition.



To this Purpoſe it muſt be remembered, that to meaſure the Greatneſs of any Angle, as A C F, the Way is, to ſet one Foot of the Compaſſes at the angular Point C, and from thence to deſcribe a Circle. Then the Arc A F of that Circle meaſures the Greatneſs of the Angle A C F; the Angle being reckoned greater er or leſs, as that Arc of the Circle intercepted between the Legs C A, C F, is greater or leſs. So the Angle D C F is greater than the Angle A C F, becauſe the Arc D F is greater than the Arc A F. If then from the Point D, where the Leg C D cuts the Circle, we let fal a Perpendicular D S to cut the other Leg C F in S, that right Line D S is called the Sine of the Angle D C F. So the Perpendicular A S is the Sine of the Angle A C S. Now from this it may be obſerved,

1. THAT as the Angle grows greater, the Sine grows greater. So the Sine D S is longer than A S, as being the Sine of a greater Angle.

2. AS it appears by Inſpection, that as the Sines grow larger, they approach nearer to the Radius C R at right Angles to C F, therefore when the Angle is increaſed to 90°, the Radius and the Sine coincide. So that the Sine of 90° is the ſame Thing with the Radius.

THE next Queſtion is then to know, what Proportion the Sines of the ſeveral Angles bear to one another. And this by the commendable Induſtry of the Geometricians, that have lived before us, has been calculated with prodigious Labour, to every Degree and Minute of the Circle, in the Tables of Natural Sines, to be met with in ſeveral Mathematical Books. It is ſufficient for our Purpoſe to take thoſe of the ſeveral Degrees only, which are ſet down in this Table.

SUPPOSING then the Radius or Sine of 90°, to conſiſt of 100000 Parts, the Sines of the other Angles will be of the ſeveral Values here exhibited.

	Deg.	Parts	Deg.	Parts	Deg.	Parts
Sine of	1	1745	31	51503	61	87461
	2	3489	32	52991	62	88294
	3	5233	33	54463	63	89100
	4	6975	34	55919	64	89879
	5	8715	35	57357	65	90630
	6	10452	36	58778	66	91354
	7	12186	37	60181	67	92050
	8	13917	38	61566	68	92718
	9	15643	39	62932	69	93358
	10	17364	40	64278	70	93969
	11	19080	41	65605	71	94551
	12	20791	42	66913	72	95105
	13	22495	43	68199	73	95630
	14	24192	44	69465	74	96126
	15	25881	45	70710	75	96592
	16	27563	46	71933	76	97029
	17	29237	47	73135	77	97437
	18	30901	48	74314	78	97814
	19	32556	49	75470	79	98162
	20	34202	50	76604	80	98480
	21	35836	51	77714	81	98768
	22	37460	52	78801	82	99026
	23	39073	53	79863	83	99254
	24	40673	54	80901	84	99452
	25	42261	55	81915	85	99619
	26	43837	56	82903	86	99756
	27	45399	57	83867	87	99862
	28	46947	58	84804	88	99939
	29	48480	59	85716	89	99984
	30	50000	60	86602	90	100000

Now by the Help of this Table, the ſeveral Propoſitions relating to the inclined Plane may be exemplified, and any Queſtions relating to that Power may be ſpeedily reſolved.

Let there be given then a Weight of 50 Pounds, rolling down an inclined Plane, whoſe Angle of Inclination is 30 deg. and let it be required to know the Power able to ſuſtain *it* upon that Plane.

By the Prop. above the Weight will be to the Power, as Radius or the Sine of *9*0° is to the Sine of the Angle of Inclination, which is in this Queſtion 30°. Look in the Table for the Sine of 30°, which is of 50000 Parts. Then ſay,

As 100000 the Sine of *9*0°, or Radius is to 50000 the Sine of 30°; ſo is 50 Pounds the Quantity of the Weight to the Power able to ſuſtain it on that Plane.

To find the Quantity of that Power, we have now nothing but a common Queſtion of the Rule of Three to reſolve. For multiplying the ſecond Number 50000 by the third, which is 50; and dividing by 100000 the firſt, the Quotient is 25, the Quantity of the Power. So that a Power equal to 25 Pounds will ſuſtain a Weight of 50 upon an inclined Plane, whoſe Angle of Inclination is 30 deg.

So if the Queſtion were inverted, and the Power 25 Pounds, and the Angle of Inclination 30° be given, and the Weight required, it is anſwered in the ſame manner, by only tranſpoſing the Terms, and ſaying, As 50000 to 25, ſo 100000 to the Weight, which, by multiplying and dividing according to the Golden Rule, comes out 50 Pounds, which is the Weight ſuch a Power is able to ſuſtain upon a Plane of that Inclination.

AND if the Power and Weight were given, and the Angle of Inclination required, it is found by diſpoſing the Terms in another Order; and ſaying, As 50 the Weight to 100000, the Sine of 90° or Radius, ſo is 25 the Power of the Sine of the Angle of Inclination. Multiply 25 by 100000, and divide by 50, and the Quotient is 50000. Look in the Table of Sines for that Number, and you will find it ſtanding over againſt 30°, which is the Angle of Inclination, neceſſary to enable a Power of 25 Pounds to ſupport a Weight of 50.

AND after this Manner, all other Queſtions of the ſame Kind are reſolved.

BY the Help of this Table, we may likewiſe anſwer all Queſtions relating to the ſecond Propoſition. As for Example. It being already known by the foregoing Prop. that a Power of 25 Pounds, applied in a Direction parallel to the Plane, will ſuſtain a Weight of 50

Pounds when the Angle of Inclination is 30°, it is required to find what Weight the ſame Power will ſuſtain when it is applied in a Direction *inclined* to the ſame Plane in an Angle of 20 Deg. or in the Words of that Propoſition, when the Angle of Application of the Power is 20 deg.

Now by that Prop. the Effect of the Power in this Direction will be to the Effect of the ſame Power in a Direction parallel to the Plane, as the Sine Complement of the Angle of Application to Radius. The Complement of any Angle is ſo much as it wants of 90 deg. To know therefore the Complement of any Angle, ſubſtract the Number expreſſing the Quantity of that Angle from 90, and the Remainder is the Complement. So in the preſent Caſe the Angle of Application of the Power is 20 deg. take 20° from 90°, the Remainder is 70° the Complement of that Angle. The Sine of 70° is therefore the Sine of the Complement of the Angle of Application, called for Brevity the Sine Complement. The Proportion will therefore be, As 100000 or Radius is to the Sine of 70°, ſo is 50 Pounds, the Weight which the Power can ſupport when applied in a Direction parallel to the Plane, to another Number. Look in the Table for the Sine of 70° which is 93969; then ſay, As 100000 to 93969, ſo is 50 to the Weight ſought; and by working by the Golden Rule, the Weight ſought will come out 46 Pounds, and $\frac{9}{10}$ of a Pound, or 47 Pounds very nearly. So that by the oblique Application of the Power, there is loſt above three Pounds in its Effect. And by the ſame Method the Loſs will appear ſtill greater, if the Angle of Application be ſtill greater.

We come now to the third Propoſition, which is, that the Preſſure of the Weight is as the Sine Complement to 90° of the Angle of Inclination; or which is the ſame thing, that the Quantity of Preſſure of the Weight upon the Plane is to the whole Weight, as the Sine

Complement to 90° of the Angle of Inclination is to Radius. Let it then be required to find what the Quantity is of the Preſſure of the abovementioned Weight of 50 Pounds on the inclined Plane aforeſaid, whoſe Angle of Inclination is 30°.

FIND firſt the Complement of 30° which is 60°. Then find in the Table the Sine of 60° which is 86602. Then the Proportion will be, As Radius or 100000 to 86602 the Sine Compl. of 30° or the Sine of 60°, ſo is the Weight of 50 Pounds to the Quantity of its Preſſure on the Plane; and this Preſſure, by working by the Golden Rule, will come forth 44 Pounds and $\frac{1}{3}$ of a Pound.

SO if it were required to find an Angle of ſuch an Inclination, as that the Preſſure of a given Weight, as 50 Pounds, ſhould be neither greater nor leſs than a Quantity aſſigned, as for Example 44 Pounds and one third; then by tranſpoſing the Terms ſay, As 50 the whole

Weight to 100000 the Radius, ſo is 44, 3 to the Sine Complement of the Angle required. Then multiplying and dividing as before, there is produced the Number 86602. Look for this Number in the Table of Sines, and it ſtands over againſt 60 deg. the Complement of the Angle ſought, which Angle, by Conſequence, muſt be 30 deg. as above.

THE fourth Propoſition is that the Force of Percuſſion or Impulſe of any Body in Motion upon a Surface, is as the Sine of the Angle of Incidence, *cæteris paribus*, that is, ſuppoſing the Velocity and Magnitude of the Body to be in all Caſes the ſame. Therefore the Table of Sines exhibits by Inſpection the Proportions of the ſeveral Impulſes at the ſeveral Degrees of Inclination. That is, a Body impinging on a Surface at one Degree of Inclination impels that Surface with 1745 Parts of Force, at two Degrees with 3489 Parts, &c. till at 90° of Inclination, or when it falls on the ſame Surface perpendicularly, it impels it with 100000 Parts of Force. By dividing one Sine by another, we may expreſs this Proportion ſtill more conciſely. As if it were required to know, how much the Impulſe at 90° of Inclination is greater than at one Degree of Inclination. Divide 100000 by 1745, and the Quotient is 57 and a Fraction, which ſhews that the Effect of a Body impinging on a Surface at one Degree of Incidence, is above 57 times leſs than the Effect of the ſame when impinging on a Surface perpendicularly. And in the ſame Manner may be found the Proportion between any other Angles.

AGAIN; ſuppoſe it were required to aſſign an Angle of Inclination, in which a Body in Motion impelling a Surface ſhould produce an Effect in a given Proportion to the Effect of the ſame Body impinging on it in another given Angle. As for Example. If it were required to aſſign an Angle which ſhould produce an Effect four times greater than an Angle of 9 deg. Firſt look in the Table for the Sine of 9 deg. and you find it 15643. Multiply this by 4, and the Product is 62572. Now this Number is not to be found in the Table; but one very near it ſtands over againſt 39 deg. *viz.* 62932. Therefore taking the Angle a little leſs than 39 deg. it will have the Effect required.

If it be neceſſary to aſſign this Angle more nicely, we muſt conſult the large Tables calculated to Minutes, where there appears 62569 over againſt 38 deg. 44 min. which is the Angle required. It is true, this Number is not exactly the ſame as the other; but the Difference is ſo ſmall, as to be of no Conſideration. In the ſame Manner one may diminiſh the Impulſe in the Proportion required, by dividing the Sine of 9 deg. by 4, and proceeding as above.

You ſee that by diminiſhing the Angle one may diminiſh the Impulſe in any Proportion required; but we can't contrarywiſe by increaſing the Angle increaſe the Effect as high as we pleaſe. The Increaſe has a Limit, which the Diminution has not. For Example. Suppoſe it were required to find an Angle, whoſe Effect ſhould be 4 Times leſs than that of an Angle of 30 deg. By dividing 50000 the Sine of the given Angle by 4, there is produced 12500, the Sine of 7 deg. 11 min. the Angle required; ſo that this Demand is anſwer'd. But if it were required to aſſign an Angle, whoſe Effect ſhould be four Times greater than that of 30 deg. this would be found impoſſible. For multiplying 50000, the Sine of 30° by 4, the Product is 200000. But the greateſt Number poſſible to ſtand in this Table is 100000, becauſe the Sine of a right Angle is the greateſt of all the Sines. And therefore in all Caſes of this Nature, when the Product exceeds 100000, it ſhews that the Effect required cannot be produced by increaſing the Angle only; but that alſo an Addition muſt be made to the Velocity or Magnitude of the Body in Motion,

tion, in Order to increaſe the Impulſe in that Proportion.

In treating of the Sail it was ſhewn that the Effect of that Power would be, as the Square of the Sine of the Angle of Incidence of the Wind upon the Sail, as the Square of the Velocity, and as the Superficies or Area of the Sail it ſelf. It may be proper to make the Meaning of all this more clear by exemplifying it in Numbers.

Suppose then the Wind acting upon two Sails; upon one in an Angle of 45°, and upon the other in an Angle of 67° 30′. Or which is the ſame thing, ſuppoſe two Ships, one in *Europe*, the other in *Aſia* (having each a ſingle Sail only) to be ſteering directly South; and that the former had the Wind in the North-Weſt Point, and the Latter in the North North-Weſt. Let the North-Weſt Wind blow twice as ſtrong as the N. N. W. and let the Sail of the former contain 20 Foot Square, and the

Sail of the latter 30 Square Feet. The Queſtion then is, how we ſhall know the Proportion of the Effect of theſe two Winds upon theſe two Ships, or how much one of theſe will ſail faſter than the other. And this Queſtion is anſwered by the Propoſition foregoing.

AND firſt, the Effect will be as the Square of the Sine of the Angle of Incidence. Therefore, ſince the Wind blowing from the N. W. makes an Angle of 45° with the Sail of a Ship that ſteers directly South; and from the N. N. W. makes an Angle of 67°, 30′, it follows that its Effect upon each will be as the Square of the Sine of 45°, and as the Square of the Sine of 67°, 30′. Look in the Table above for the Sine of 45°, which is 70710. Multiply this into itſelf, and it produces the Square of the Sine, which is 4999904100. In like Manner find the Sine of 67°, 30′ from the large Tables of Degrees and Minutes, which is 92387. This multiplied into it ſelf, produces its Square 85353.

57769. Therefore the N. W. Wind drives the Ship forwards with a Force, that is, as 4999904100; and the N. N. W. with a Force, that is, as 8535357769. Cut off an equal Number of Figures in both of them, and the Velocity of the firſt Ship will be to that of the latter, as 499 to 853, which is ſomething more than half ſo great.

But this is ſo ſuppoſing the Velocity or Strength of the Wind to be equal in both Caſes. But ſince the N. W. Wind was ſuppoſed to blow twice as ſtrong as the N. N. W. this will produce a new Conſequence. For the Velocity of the former being as 2, and of the other as 1, the Squares of the Velocities will be 2 multiplied by 2, and 1 multiplied by 1, that is 4 and 1. Therefore to find the true Proportion of the Velocities of theſe two Ships, we muſt multiply the Numbers before found, *viz.* 499 and 853, ſeverally into the Squares of the Velocity of the Wind reſpectively belonging to each; that is 499 by 4, and

853 by 1, which produce 1996, and 853; ſo that the former Ship will ſail with a Velocity, as 199, and the ſecond with a Velocity, as 85; which is as 2 and $\frac{1}{3}$ to 1.

BUT this is ſtill ſuppoſing the Sails to be equal in both Caſes; but theſe being ſuppoſed different, it will be neceſſary to multiply the two Numbers laſt found, ſeverally by the Areas of the reſpective Sails. Multiplying therefore 1996 by 20 the Area of the Sail of the former Ship, and 853 by 30 the Area or Superficies of the Sail belonging to the latter; the Products will be 39920 and 25590; that is, the Velocities of theſe two Ships, ariſing from the different Circumſtances here ſuppoſed, will be to each other, as 39920 to 25590, or by a more conciſe Expreſſion, as 39 to 25, or as 3 to 2.

IT may not be amiſs to obſerve that, when the Angles in a Queſtion are expreſſde in odd Minutes, the Sines of thoſe Angles may be collected very nearly ly from the Table above that anſwers only to Degrees. As if it be required from that Table to find the Sine 87°, 21′. Take the Sine of 87°, which is *99862*, and the Sine of 88°, which is *99939*; ſubſtract the leaſt from the greateſt, and the Remainder is 77. Divide this by 60, and the Quotient is 1. Multiply this Quotient by the Number of odd Minutes, which is 21, which produces 21. Add this Number ſo found to the Sine of 87, and it gives the Sine of 87°, 21′ very nearly. For this Addition produces the Number *99883*; and the true Sine is *99893*; ſo that the Difference between the Sine ſo found and the true one is but of one Part in 10000.

FINIS.

Errors of the Preſs in Mathematical Treatiſes being particularly troubleſome, the Reader is deſired to take Notice of theſe that follow.

ERRATA.

Pag. 8. lin. 6. from the Bottom for *is to* read *into*. Pag. 24. lin. penult. read *a conſpiring or contrary*, &c. Pag. 29. lin. 19. add, *and the Force which acts, as A E to A F*. Pag. 48. lin. 16. read *ſhorter or longer*. Ibid. lin. 18. read *leſs or greater*. Pag. 54. In the Scheme put in the Letter *D*, in the Line *E C D*. Pag. 56. add the Letter *D* in the Scheme to the Weight on the left Hand. Pag. 66. for *this* read *his*. Pag. 110. lin. 10. read 2*d. Fig*. Pag. 111. lin. 3. from the Bottom read *with*. Pag. 192. lin. 2. read *Fig.* 1. &c. Pag. 193. lin. 13. read, *in the 2d Figure*, &c.

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